Solution

MATRICES UNIT TEST

Class 12 - Mathematics

1. **(a)** nk

   Explanation:

   $\because A = [a_{ij}]_{n\times n}$

   Trace of A, i.e., tr (A) = $\sum a_{ij}^{n} i$ = 1 = $a_{11} + a_{22}$ + ... + a_{nm}

   = k + k + k + k + k + ...(n times)

   = k(n)

   = nk

2. **(a)** 6

   Explanation:

   Since the matrix has 18 elements.

   Therefore, following are the possible orders

   $1\times18, 18\times1, 2\times9, 9\times2, 3\times6, 6\times3$

   Hence, the number of possible orders are 6.

3.

   (b) 9

   Explanation:

   9

4.

   (d) 64

   Explanation:

   The order of the matrix = 2×3

   The number of elements = 2×3 = 6

   Each place can have either 1 or 2. So, each place can be filled in 2 ways.

   Thus, the number of possible matrices = 2^6 = 64

5. (a) - (iv), (b) - (i), (c) - (ii), (d) - (iii)
6. Since $a_{ij} = e^{2ix}$ sin jx so, for i=1, j=2,

   we have, $a_{12} = e^{2x}$ sin 2x.
7. The information is represented in the form of a 3×2 matrix as follows

   $$A=\begin{bmatrix}30 & 25\\ 25 & 31\\ 27 & 26\end{bmatrix}$$

   The entry in the third row and the second column represents the number of women workers in factory III.
8. Order of matrix A is 2×2
9. Since, a matrix having mn element is of order m × n.
   1. Therefore, there are 8 possible matrices having 24 elements of orders 1 × 24, 2 × 12, 3 × 8, 4 × 6, 24 × 1, 12 × 2, 8 × 3, 6 × 4.
   2. Prime number 13 = 1 × 13 and 13 × 1. Therefore, there are 2 possible matrices of order 1 × 13 (Row matrix) and 13 × 1 (Column matrix).
10. (a) - (iii), (b) - (iv), (c) - (i), (d) - (ii)
11. (a) - (ii), (b) - (iv), (c) - (i), (d) - (iii)
12. $A^{-1} = \frac{adjA}{|A|}$

    $|A|$ = 5(-1) + 4(1) = -1

    $C_{11} = -1, C_{21} = 8, C_{31} = -12$

    $C_{12} = 0, C_{22} = 1, C_{32} = -2$

$C_{13} = 1, C_{23} = -10, C_{33} = 15$

$A^{-1} = \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$

$(AB)^{-1} = B^{-1} A^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$

$(AB)^{-1} = \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$

13. Given that A is a symmetric matrix

$\therefore A = A^T \ldots$(i)

Now, we have to check A^n is symmetric or skew-symmetric

$(A^n)^T = (A \times A \times A \times A \ldots A)^T$ [for all $n \in N$]

$\Rightarrow (A^n)^T = (A^T \times A^T \ldots A^T)$

$[\because (AB)^T = B^T A^T]$

$= A \times A \ldots A$ [from (i)]

$= A^n$

$\Rightarrow (A^n)^T = A^n$

For all natural numbers

So, A^n is a symmetric matrix

14. We must understand what is the nature of a symmetric matrix is.

A symmetric matrix is a square matrix that is equal to its transpose.

A symmetric matrix $\Leftrightarrow A = A^T$(based on the property of the transpose of a matrix)

Now, let us understand what is the nature of a skew-symmetric matrix is.

A skew-symmetric matrix is a square matrix whose transpose equals its negative, that, it satisfies the condition

A skew-symmetric matrix $\Leftrightarrow A^T = -A$ (based on the property of the transpose of a matrix)

And,

A square matrix is a matrix with the same number of rows and columns. An n-by-n matrix is known as a square matrix of order n.

We need to find a square matrix that is both symmetric as well as skew-symmetric.

Take a 2×2 null matrix.

Say,

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Let us take the transpose of matrix A.

We know that the transpose of a matrix is a new matrix whose rows are the columns of the original.

So,

$A^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Since, $A = A^T$.

$\therefore$ A is symmetric.

Take the same matrix and multiply it with -1.

$-A = -1 \times \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow -A = -\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow -A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Let us take the transpose of the matrix –A.

So,

$-A^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Since,

A^T = -A

∴, A is skew-symmetric.

Thus, A (a null matrix) is both symmetric as well as skew-symmetric.

15. Given: A = [a_{ij}] is a square matrix such that $a_{ij} = i^2 - j^2$

Suppose A is a 2 × 2 square matrix ,then the entries of the matrix can be as follows.

A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Here,given that

$a_{ij} = i^2 - j^2$ Using this relationship between the i and j ,the following values of the elements of the matrix are obtained.

So, $a_{12} = (1)^2 - (2)^2 = 1 - 4 = -3$

and $a_{21} = (2)^2 - (1)^2 = 4 - 1 = 3$

For diagonal elements, i = j, we have

$a_{11} = (1)^2 - (1)^2 = 0$

and $a_{22} = (2)^2 - (2)^2 = 0$

So, Matrix A becomes

A = $\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$

Now, we have to check A is symmetric or skew-symmetric.

We know that, if a matrix is symmetric then A^T = A

and if a matrix is skew-symmetric then A^T = -A

So, firstly we find the A^T(means the transpose of A)

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2 × 2 matrix, then the transpose of a matrix is $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

So,

$A^T = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

$\Rightarrow A^T = -\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$

$\Rightarrow A^T = -A$

∴ A is a skew-symmetric matrix.