

# Matrices

---

## Question 25: A Matrix that is Both Symmetric and Skew-Symmetric

- A symmetric matrix satisfies:

$$A^T = A$$

- A skew-symmetric matrix satisfies:

$$A^T = -A$$

- If a matrix is both symmetric and skew-symmetric, then:

$$A = -A \Rightarrow 2A = 0 \Rightarrow A = 0$$

- The only possibility is a **null matrix (zero matrix)**.
  - **Correct answer: (b) Null**
- 

## Question 26: Conditions for $AB = A$ and $BA = B$

- Given:

$$AB = A, \quad BA = B$$

- Squaring both sides:

$$B^2 = BAB = B$$

- This means  $B$  is **idempotent** ( $B^2 = B$ ).
  - **Correct answer: (b) B**
- 

## Question 27: Squaring $(A + B)$ under Given Condition

- Given:

$$B = -A^{-1}BA$$

- Expanding:

$$(A + B)^2 = A^2 + AB + BA + B^2$$

- Substituting  $BA = -A^{-1}BAA = -AB$ , we get:

$$(A + B)^2 = A^2 + 2AB + B^2$$

- **Correct answer: (c)  $A^2 + 2AB + B^2$**
-

### Question 28: Power of a Diagonal Matrix

- Given:

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

- For a **diagonal matrix**, raising it to a power means raising each element:

$$A^n = \begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$$

- Correct answer: (c)  $A^n$  with element-wise exponentiation**
- 

### Question 29: Matrix Properties

- Given:

$$A = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$$

- Checking each statement:

- $AA^T = I \rightarrow$  False (Compute  $AA^T$ )
- $BB^T = I \rightarrow$  False (Compute  $BB^T$ )
- $AB \neq BA \rightarrow$  True (Matrix multiplication is not necessarily commutative)
- $(AB)^T = I \rightarrow$  False (Transpose doesn't necessarily yield identity)

- Correct answer: (c)  $AB \neq BA$**
- 

### Question 30: Identifying a Skew-Symmetric Expression

- A **skew-symmetric matrix** satisfies:

$$A^T = -A$$

- Checking each option:

- $A + A^T \rightarrow$  Symmetric, not skew-symmetric.
- $A - A^T \rightarrow$  Skew-symmetric.
- $AA^T \rightarrow$  Not necessarily skew-symmetric.
- $A^T A - A \rightarrow$  Not necessarily skew-symmetric.

- Correct answer: (b)  $A - A^T$**
- 

### Key Concepts Covered in These Questions:

- Symmetric and Skew-Symmetric Matrices:** Understanding their properties and conditions.
- Matrix Idempotency:** If  $B^2 = B$ , then  $B$  is idempotent.
- Matrix Exponentiation:** Power of diagonal matrices.

4. **Matrix Multiplication Properties:** Commutativity of matrices.

5. **Matrix Transformations:** Square roots, transposes, and inverse conditions.

These are crucial concepts in **linear algebra** and have applications in **engineering, physics, and computational mathematics**.