Matrices

Question 15: Transformation of a Given Matrix

- The given matrix A is:

$$A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$$

- The transformation involves computing $(I - A)^{-1}$.
- Using matrix properties, the result is:

$$(I - A)^{-1} = I + A$$

- Correct answer: **(a)** $I + A$.

Question 16: Decomposition into Symmetric and Skew-Symmetric Matrices

- A matrix can be expressed as:

$$A = B + C$$

  where B is symmetric ($B^T = B$) and C is skew-symmetric ($C^T = -C$).
- The symmetric part is:

$$B = \frac{A + A^T}{2}$$

- The skew-symmetric part is:

$$C = \frac{A - A^T}{2}$$

- Computing these values, we obtain **option (c)**.

Question 17: Square Matrix with $A^2 = I$

- Given $A^2 = I$, it follows that $A^{-1} = A$.
- Expanding the given expression:

$$(A - I)^3 + (A + I)^3 - 7A$$

- Using matrix polynomial simplifications, the correct result is **(c)** $I + A$.

Question 18: Idempotent Matrix Property

- A matrix B is idempotent if:

$$B^2 = B$$

- Setting $A = I - B$, we can derive:
  - $A^2 = I - 2B + B^2$
  - Since $B^2 = B$, we simplify to $A^2 = I$.
- Correct answer: **(a)** $A^2 = I$.

Question 19: Multiplication of Rotation Matrices

- Given:

$$F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- This represents a **3D rotation matrix**.
- The multiplication follows:

$$F(\alpha) \cdot F(\beta) = F(\alpha + \beta)$$

- Correct answer: **(c)** $F(\alpha + \beta)$.

Question 20: Matrix Function $A(x)$

- Given:

$$A(x) = (1 - x) \begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix}$$

- The transformation conditions simplify to:

$$A(z) = A(x)A(y)$$

- Correct answer: **(c)** $A(z) = A(x)A(y)$.

Question 21: Power of a Given Matrix

- Given:

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

- This represents a **90-degree rotation matrix**.
- Raising to the 16th power follows a cyclic pattern:

$$A^4 = I, \quad A^{16} = (A^4)^4 = I$$

- The correct answer is **(d)** I.

Question 22: Square Root of the Identity Matrix

- Given:

$$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

  and we need to find conditions for it to be a square root of the identity matrix.
- Squaring the matrix and setting it equal to I leads to:

$$\alpha^2 + \beta\gamma = 1$$

- Correct answer: **(d)** $\alpha^2 + \beta\gamma = 1$.

Question 23: Matrix Commutator Property

- The given expression:

$$AB' - BA'$$

  represents a **commutator**.
- This structure leads to a **skew-symmetric matrix**.
- Correct answer: **(a) skew-symmetric matrix**.

Question 24: Solving for Matrix A

- Given equation:

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- This means the given matrix is the **inverse of the product**.
- Solving leads to:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Correct answer: **(b)** I.

These questions test concepts related to:

- **Matrix transformations**
- **Rotation matrices**
- **Idempotent and inverse matrices**
- **Commutator properties**
- **Matrix exponentiation**

Each question explores a fundamental matrix property, making it essential for mastering **linear algebra**.