Matrices

Question 5: AA^T Properties

- AA^T represents a matrix multiplied by its transpose.
- This always results in a **symmetric matrix** because:

$$(AA^T)^T = A^{TT}A^T = AA^T$$

- Hence, the answer is **(b) symmetric matrix**.

Question 6: Product of Rotation Matrices

- The given matrix:

$$f(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

  is a **rotation matrix**.
- The product $f(\alpha)f(\beta)f(\gamma)$ represents **three successive rotations**.
- If α, β, γ are angles of a triangle, then:

$$\alpha + \beta + \gamma = \pi$$

- The composition of these three rotations results in **a full rotation back to identity or negative identity matrix**.
- The answer depends on the determinant condition, which leads to **(b)** $-I_2$.

Question 7: Properties of Matrices

- (a) $AB = AC$ does **not** imply $B = C$ unless A is invertible.
- (b) The given polynomial equation suggests that A satisfies a **Cayley-Hamilton theorem** form, making A invertible.
- (c) If $A^2 = 0$, then it does not necessarily mean $A = 0$, only that A is **nilpotent**.
- The correct statement is **(b) A is invertible**.

Question 8: Rotation Matrix Properties

- The given matrix A_α is another form of a **rotation matrix**.
- The multiplication property states:

$$A_\alpha A_{-\alpha} = I$$

  meaning rotation by α followed by $-\alpha$ results in identity.
- The correct answer is **(a)** $A_\alpha A_{-\alpha} = I$.

Question 9: Finding A^{-1} Given a Factorization

- The given condition:

$$(A - 2I)(A + I) = 0$$

  means that A satisfies a quadratic equation.
- To find A^{-1}, rewrite it as:

$$A^{-1} = \frac{A + I}{2}$$

- Correct answer: **(b)** $\frac{A+I}{2}$.

Question 10: Properties of a Square Matrix P

- Given $P^2 = I - P$, solve for P^n using recursive relations.
- If $P = 5I - 8P$, then by substituting and solving powers iteratively, we find $n = 5$.
- Correct answer: **(b) 5**.

Question 11: Transformation of P and Q

- Given the matrix transformations, the key concept here is **similarity transformation**:

$$Q = PAP^T$$

- The exponentiation of Q follows a pattern, and simplifying the result leads to **option (c)**.

Question 12: Matrix Polynomial Computation

- The given matrix:

$$A^2 + 2A^4 + 4A^6$$

  follows a characteristic equation approach.
- Using matrix powers and summing terms, the result simplifies to $8I$.
- Correct answer: **(c)** $8I$.

Question 13: Commutativity of Two Matrices

- Two matrices **commute** if $AB = BA$.
- Given:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

  their commutativity condition leads to certain restrictions on a and b.
- It is found that there exist **a finite number of such B matrices**.

- Correct answer: **(b)**.

Question 14: Eigenvalue Relation with Cube Roots of Unity

- The matrix involves **cube roots of unity**:

$$\omega = e^{2\pi i/3}, \quad \omega^2 = e^{4\pi i/3}$$

- The determinant computation results in $k = 6$.
- Correct answer: **(a) 6**.

These questions cover key **matrix concepts**, including **matrix powers, determinants, eigenvalues, invertibility, nilpotency, and rotation matrices**.