Matrices

Here's a breakdown of the concepts behind the given questions in the image:

Question 1: Finding n in $B^n - A = I$

- Given matrices A and B, the equation $B^n - A = I$ implies:

$$B^n = A + I$$

- This requires computing the powers of B to determine when it matches $A + I$.
- The key concept here is matrix exponentiation and understanding how matrix powers behave.

Question 2: Computing A^{100} for a Special Matrix

- The given matrix is:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

- Notice that this is a rank-1 matrix where all elements are the same.
- Using matrix exponentiation properties:

$$A^n = 2^{(n-1)} A$$

- This follows from the fact that A has eigenvalues 2 and 0.
- The answer follows from repeated multiplication.

Question 3: Condition for $A^T + A = I_2$

- The matrix is a **rotation matrix**:

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

- The condition $A^T + A = I_2$ expands to:

$$\begin{bmatrix} 2\cos\theta & 0 \\ 0 & 2\cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

  which simplifies to:

$$2\cos\theta = 1 \Rightarrow \cos\theta = \frac{1}{2}$$

- Solving for θ, the general solution is $\theta = (2n+1)\pi/3$, giving conditions on θ.

Question 4: Computing $(aI + bA)^2$

- The given matrix:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

- Key property:

$$A^2 = 0$$

  since multiplying it by itself gives a zero matrix.
- Expanding the expression:

$$(aI + bA)^2 = a^2I + abA + abA + b^2A^2$$

- Since $A^2 = 0$, the last term vanishes:

$$a^2I + 2abA$$

- The correct choice is $a^2I + 2abA$.

These questions test fundamental **matrix operations**, including **matrix exponentiation, properties of rotation matrices, and nilpotent matrices**.