Matrices and Determinants - Matrix Operations and Exponentiation

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Question 25: Inverse of a Rotation Matrix

- The given matrix:

$$F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

  is a **3D rotation matrix**.
- The inverse of a rotation matrix is its **transpose**:

$$F(\alpha)^{-1} = F(-\alpha)$$

- **Correct answer: (a)** $F(-\alpha)$.

Question 26: Condition for a Non-Trivial Solution in a Linear System

- Given equations:

$$x\sin 3\theta - y + z = 0, \quad x\cos 2\theta + 4y + 3z = 0, \quad 2x + 7y + 7z = 0$$

- For a **non-trivial solution**, the determinant must be **zero**.
- Solving, the condition on θ is:

$$\theta = \pi\left(n + \frac{(-1)^n}{3}\right)$$

- **Correct answer: (a)** $\pi\left(n + \frac{(-1)^n}{3}\right)$.

Question 27: Adjugate of a Matrix Expression

- Given conditions:

$$\text{adj}(B) = A, \quad |\mathbf{P}| = 1, \quad |\mathbf{Q}| = 1$$

- Using properties of the **adjugate matrix**, we derive:

$$\text{adj}(Q^{-1}BP^{-1}) = PAQ$$

- **Correct answer: (c) PAQ**.

Question 28: Finding Valid x Values in a System of Equations

- Given the system:

$$\lambda x + (\sin \alpha)y + (\cos \alpha)z = 0$$

$$x + (\cos \alpha)y + (\sin \alpha)z = 0$$

$$-x + (\sin \alpha)y - (\cos \alpha)z = 0$$

- The system has a **non-trivial solution** if the determinant of the coefficient matrix is **zero**.
- Solving, we find possible values:

$$x \in \{-\sqrt{2}, 0\}$$

- **Correct answer: (b)** $[-\sqrt{2}, 0]$.

Question 29: Inverse of a Diagonal Matrix

- Given:

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

- The inverse of a diagonal matrix is another diagonal matrix:

$$A^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$$

- **Correct answer: (a) Statement 1 is true, Statement 2 is true, and Statement 2 explains Statement 1.**

Question 30: Condition for Concurrent Lines

- The given system represents three lines:

$$L_1 : a_1x + b_1y + c_1 = 0$$

$$L_2 : a_2x + b_2y + c_2 = 0$$

$$L_3 : a_3x + b_3y + c_3 = 0$$

- For the three lines to be **concurrent**, the determinant:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

  must hold.
- **Statement 1 is correct, Statement 2 is incorrect** because determinant zero does not necessarily imply concurrency in general cases.
- **Correct answer: (b) Statement 1 is true, Statement 2 is false.**

Key Concepts Covered:

1. **Rotation Matrices**: How their inverse is their transpose.
2. **Linear Systems & Determinants**: Conditions for non-trivial solutions.
3. **Adjugate Matrices**: How they interact with matrix transformations.
4. **Diagonal Matrices**: Their inverse is also diagonal.
5. **Concurrency of Lines**: Determinant condition for intersection.

These topics are crucial in **linear algebra, geometry, and engineering mathematics**.