Matrices and Determinants - Matrix Operations and Exponentiation

---

Question 4: Properties of a Triangle Determinant

- The determinant is given as:

$$\begin{vmatrix} \sin A \sin^2 A & 1 \\ \sin B \sin^2 B & 0 \\ \sin C \sin^2 C & 1 \end{vmatrix}$$

- The determinant vanishes if the given matrix represents a **dependent system**.
- The result classifies the triangle as **right-angled or isosceles**.
- **Correct answer: (c) Right-angled or isosceles**

Question 5: Determinant with Floor Function

- The determinant is:

$$\begin{vmatrix} \lfloor x \rfloor + 1 & \lfloor y \rfloor & \lfloor z \rfloor \\ \lfloor x \rfloor & \lfloor y \rfloor + 1 & \lfloor z \rfloor + 1 \\ \lfloor x \rfloor & \lfloor y \rfloor & \lfloor z \rfloor + 1 \end{vmatrix}$$

- Expanding the determinant leads to simplifications.
- The final value depends on **integer properties of the floor function**.
- **Correct answer: (a)** $\lfloor z \rfloor$.

Question 6: Nature of a Determinant with Complex Numbers

- The determinant involves complex conjugates:

$$\begin{vmatrix} 0 & -y & -z \\ \overline{y} & 0 & -x \\ \overline{z} & \overline{x} & 0 \end{vmatrix}$$

- Using properties of **Hermitian matrices**, the determinant is purely **imaginary**.
- **Correct answer: (b) Purely imaginary**

Question 7: Infinite Solutions of a System

- The system:

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 6 \\ x_1 + 3x_2 + 5x_3 = 9 \\ 2x_1 + 5x_2 + ax_3 = b \end{cases}$$

- For infinite solutions, the determinant must be **zero** (rank condition).
- Solving for a and b, we get $a = 8$ and $b = 15$.
- **Correct answer: (d)** $a = 8, b = 15$.

Question 8: Determinant Vanishing Condition

- The determinant:

$$\begin{vmatrix} 1 & (x-3) & (x-3)^2 \\ 1 & (x-4) & (x-4)^2 \\ 1 & (x-5) & (x-5)^2 \end{vmatrix}$$

  represents a **Vandermonde determinant**.
- The determinant is **zero when x matches any row value**, giving three roots.
- **Correct answer: (a) 3 values of x**.

Question 9: Summation of Determinants

- Given:

$$\Delta_r = m^2 - 1, \quad 2^m, \quad m + 1$$

- The sum over all values gives a computed result.
- **Correct answer: (d) 1**

Question 10: Determinant Sign Based on Inequality

- The determinant:

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix}$$

- Given $b^2 - ac < 0$, the determinant must be **negative**.
- **Correct answer: (c) Negative**

Question 11: Determinant Power Computation

- Given:

$$A = \begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}$$

- Computing:

$$|A^{2013} - 3A^{2012}|$$

using **recurrence relations**, we get:

$$\pm 8$$

- **Correct answer: (a) 8**.

Question 12: Maximum Value of a Determinant

- The given determinant:

$$\begin{vmatrix} \sin^2\alpha & \sin\alpha\cos\alpha & \cos^2\alpha \\ \sin^2\beta & \sin\beta\cos\beta & \cos^2\beta \\ \sin^2\gamma & \sin\gamma\cos\gamma & \cos^2\gamma \end{vmatrix}$$

  is evaluated using **trigonometric properties**.
- The determinant is at most **1**.
- **Correct answer: (a) 1**.

Question 13: Distinct Roots of a Trigonometric System

- Given:

$$\begin{cases} \sin x \cos x = 0 \\ \cos x \sin x = 0 \end{cases}$$

- The roots in the given interval are computed.
- **Correct answer: (d) 3**.

Question 14: Finding Consistency Condition

- Given:

$$p^3x + (p+1)^3y = (p+2)^3$$

$$px + (p+1)y = p+2$$

$$x + y = 1$$

- Consistency conditions are checked using **rank** and **determinants**.
- The system is consistent for $p = 0$.
- **Correct answer: (a)** $p = 0$.

Key Concepts Covered:

1. **Triangle Classification Using Determinants**: Determinants help classify geometric properties.
2. **Floor Function in Determinants**: Understanding integer properties in matrices.
3. **Complex Matrices**: How determinants behave with complex conjugates.
4. **Linear Equation Systems**: Infinite solutions and determinant conditions.
5. **Vandermonde Determinants**: Special case where determinant vanishes.
6. **Determinant Properties in Trigonometry**: How trigonometric identities simplify determinants.

7. **Consistency of Equations**: How determinants predict solvability.

These concepts are **fundamental in linear algebra** and are widely applied in **physics, engineering, and pure mathematics**.