Matrices and Determinants - Matrix Operations and Exponentiation

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Question 1: Determinant of a Trigonometric Matrix

- The given determinant involves trigonometric functions:

$$\begin{vmatrix} \cos^2 54^\circ & \cos^2 36^\circ & \cot 135^\circ \\ \sin^2 53^\circ & \cot 135^\circ & \sin^2 37^\circ \\ \cot 135^\circ & \cos^2 25^\circ & \cos^2 65^\circ \end{vmatrix}$$

- Key observations:
  - $\cot 135^\circ = -1$.
  - Using trigonometric identities, determinant properties, and row operations, the determinant simplifies.
  - The determinant evaluates to **0**, indicating that the rows are linearly dependent.
- **Correct answer: (c) is equal to 0**

Question 2: Determinant and Root Divisibility

- Given a **repeated root** a of the quadratic equation $f(x) = 0$, we analyze:

$$\begin{vmatrix} A(x) & B(x) & C(x) \\ A(a) & B(a) & C(a) \\ A'(a) & B'(a) & C'(a) \end{vmatrix}$$

- Since $A(x), B(x), C(x)$ are polynomials of degrees 3, 4, and 5 respectively, their determinant is divisible by $f(x)$, meaning:
  - The determinant has $(x - a)$ **as a factor**.
  - Given a is a **repeated root**, the determinant is **divisible by $f(x)$ for all x**.
- **Correct answer: (a) is divisible by $f(x)$ for all x.**

Question 3: Solution of a System of Linear Equations Using Determinants

- Given the system:

$$a_1x + b_1y + c_1z + d_1 = 0$$

$$a_2x + b_2y + c_2z + d_2 = 0$$

$$a_3x + b_3y + c_3z + d_3 = 0$$

  we define the determinant:

$$\Delta(a,b,c) = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

- If $\Delta(a,b,c) \neq 0$, Cramer's rule gives the solution for x:

$$x = \frac{\Delta(b,c,d)}{\Delta(a,b,c)}$$

- This follows from replacing the first column with the constant terms and computing the determinant.
- **Correct answer: (a)** $\frac{\Delta(b,c,d)}{\Delta(a,b,c)}$.

Key Concepts Covered:

1. **Determinants and Linear Dependence**: Determinants evaluate whether matrix rows are linearly dependent.
2. **Divisibility of Determinants**: If a function has a repeated root, it impacts determinant properties.
3. **Cramer's Rule**: A method to solve a system of equations using determinants.

These concepts are fundamental in **linear algebra** and are widely used in **solving equations, engineering applications, and physics**.