

## Solve in steps

(10)

$$\frac{x + 2}{(x + 1)(x^2 + 1)}$$

Let

$$\frac{x + 2}{(x + 1)(x^2 + 1)} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 + 1}$$

Multiply both sides by  $(x + 1)(x^2 + 1)$ :

$$x + 2 = A(x^2 + 1) + (Bx + C)(x + 1)$$

Expand:

$$x + 2 = Ax^2 + A + Bx^2 + Bx + Cx + C$$

Combine terms:

$$x + 2 = (A + B)x^2 + (B + C)x + (A + C)$$

Now equate coefficients:

$$\begin{cases} A + B = 0 \\ B + C = 1 \\ A + C = 2 \end{cases}$$

Solve:

From (1):  $B = -A$

Substitute into (2):  $-A + C = 1 \Rightarrow C = 1 + A$

Substitute into (3):  $A + (1 + A) = 2 \Rightarrow 2A + 1 = 2 \Rightarrow A = \frac{1}{2}$

Then  $B = -\frac{1}{2}$ ,  $C = \frac{3}{2}$ .

**Hence:**

$$\frac{x + 2}{(x + 1)(x^2 + 1)} = \frac{1/2}{x + 1} + \frac{(-\frac{1}{2}x + \frac{3}{2})}{x^2 + 1}$$

(11)

$$\frac{7x^2 - 25x + 6}{(x^2 - 2x - 1)(3x - 2)}$$

Let

$$\frac{7x^2 - 25x + 6}{(x^2 - 2x - 1)(3x - 2)} = \frac{Ax + B}{x^2 - 2x - 1} + \frac{C}{3x - 2}$$

Multiply both sides by the denominator:

$$7x^2 - 25x + 6 = (Ax + B)(3x - 2) + C(x^2 - 2x - 1)$$

Expand:

$$7x^2 - 25x + 6 = 3Ax^2 - 2Ax + 3Bx - 2B + Cx^2 - 2Cx - C$$

Combine:

$$7x^2 - 25x + 6 = (3A + C)x^2 + (-2A + 3B - 2C)x + (-2B - C)$$

Equate coefficients:

$$\begin{cases} 3A + C = 7 \\ -2A + 3B - 2C = -25 \\ -2B - C = 6 \end{cases}$$

From (3):  $C = -2B - 6$

Substitute in (1):  $3A - 2B - 6 = 7 \Rightarrow 3A - 2B = 13$

Substitute in (2):  $-2A + 3B - 2(-2B - 6) = -25 \Rightarrow -2A + 3B + 4B + 12 = -25$   
 $-2A + 7B = -37 \Rightarrow 2A = 7B + 37$

Substitute in (1):

$$3\left(\frac{7B+37}{2}\right) - 2B = 13 \Rightarrow \frac{21B+111-4B}{2} = 13 \Rightarrow 17B + 111 = 26 \Rightarrow 17B = -85 \Rightarrow B = -5$$

Then  $C = -2(-5) - 6 = 10 - 6 = 4$

Then  $A = \frac{7B+37}{2} = \frac{7(-5)+37}{2} = \frac{-35+37}{2} = 1$ .

**Hence:**

$$\frac{7x^2 - 25x + 6}{(x^2 - 2x - 1)(3x - 2)} = \frac{x - 5}{x^2 - 2x - 1} + \frac{4}{3x - 2}$$

**(12)**

$$\frac{x^2 + x + 1}{x^2 + 2x + 1}$$

Denominator:  $(x + 1)^2$ .

Let

$$\frac{x^2 + x + 1}{(x + 1)^2} = A + \frac{B}{x + 1} + \frac{C}{(x + 1)^2}$$

Multiply both sides by  $(x + 1)^2$ :

$$x^2 + x + 1 = A(x + 1)^2 + B(x + 1) + C$$

Expand:

$$x^2 + x + 1 = A(x^2 + 2x + 1) + Bx + B + C$$

Simplify:

$$x^2 + x + 1 = Ax^2 + (2A + B)x + (A + B + C)$$

Equate coefficients:

$$\begin{cases} A = 1 \\ 2A + B = 1 \\ A + B + C = 1 \end{cases}$$

From (1):  $A = 1$

From (2):  $2(1) + B = 1 \Rightarrow B = -1$

From (3):  $1 - 1 + C = 1 \Rightarrow C = 1$

**Hence:**

$$\frac{x^2 + x + 1}{(x + 1)^2} = 1 - \frac{1}{x + 1} + \frac{1}{(x + 1)^2}$$

**✓ Final Answers**

(10)	$\frac{x + 2}{(x + 1)(x^2 + 1)} = \frac{1/2}{x + 1} + \frac{-\frac{1}{2}x + \frac{3}{2}}{x^2 + 1}$
(11)	$\frac{7x^2 - 25x + 6}{(x^2 - 2x - 1)(3x - 2)} = \frac{x - 5}{x^2 - 2x - 1} + \frac{4}{3x - 2}$
(12)	$\frac{x^2 + x + 1}{(x + 1)^2} = 1 - \frac{1}{x + 1} + \frac{1}{(x + 1)^2}$