Antiderivative by Method of Inspection: Step-by-Step Explanation

The **method of inspection** is a technique for finding the antiderivative (or indefinite integral) of a function by recognizing a pattern or directly guessing the function whose derivative would give the given function. It's a quick approach when you can spot the structure of the derivative of a known function. Let's walk through this with some examples.

Steps for Finding Antiderivative by Inspection:

1. **Identify the form** of the function: Look at the function and see if it resembles the derivative of a common function.

2. **Guess the antiderivative**: Based on the structure of the given function, make an educated guess about its antiderivative.

3. **Check by differentiation**: Differentiate your guessed function to ensure it gives the original function.

4. **Include the constant of integration**: Since antiderivatives are not unique, always include the constant C.

Example 1: $\int 2x\,dx$

1. **Identify the form**: The given function $2x$ looks like the derivative of x^2. The derivative of x^2 is $2x$.

2. **Guess the antiderivative**: Based on the observation, we guess that the antiderivative is x^2.

3. **Check by differentiation**: Differentiate x^2:

$$\frac{d}{dx}(x^2) = 2x$$

   This matches the original function.

4. **Include the constant of integration**: The antiderivative is:

$$\int 2x\,dx = x^2 + C$$

Example 2: $\int e^{3x}\,dx$

1. **Identify the form**: The function e^{3x} resembles the derivative of e^{3x}, but we need to account for the chain rule.

2. **Guess the antiderivative**: If you differentiate e^{3x}, you get $3e^{3x}$, which means the antiderivative should be $\frac{1}{3}e^{3x}$.

3. **Check by differentiation**: Differentiate $\frac{1}{3}e^{3x}$:

$$\frac{d}{dx}\left(\frac{1}{3}e^{3x}\right) = e^{3x}$$

   This matches the original function.

4. **Include the constant of integration**: The antiderivative is:

$$\int e^{3x}\,dx = \frac{1}{3}e^{3x} + C$$

Example 3: $\int \cos(2x)\,dx$

1. **Identify the form**: The function $\cos(2x)$ resembles the derivative of $\sin(2x)$, but we need to account for the chain rule.

2. **Guess the antiderivative**: The antiderivative of $\cos(2x)$ should be $\frac{1}{2}\sin(2x)$, since differentiating $\sin(2x)$ gives $2\cos(2x)$.

3. **Check by differentiation**: Differentiate $\frac{1}{2}\sin(2x)$:

$$\frac{d}{dx}\left(\frac{1}{2}\sin(2x)\right) = \cos(2x)$$

   This matches the original function.

4. **Include the constant of integration**: The antiderivative is:

$$\int \cos(2x)\,dx = \frac{1}{2}\sin(2x) + C$$

Example 4: $\int \frac{1}{x}\,dx$

1. **Identify the form**: The function $\frac{1}{x}$ is a standard form and resembles the derivative of $\ln|x|$.

2. **Guess the antiderivative**: Based on the observation, the antiderivative should be $\ln|x|$.

3. **Check by differentiation**: Differentiate $\ln|x|$:

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$$

   This matches the original function.

4. **Include the constant of integration**: The antiderivative is:

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

Example 5: $\int xe^{x^2}\, dx$

1. **Identify the form**: The function xe^{x^2} resembles the derivative of e^{x^2}, as the derivative of x^2 is $2x$.

2. **Guess the antiderivative**: Since the derivative of x^2 is $2x$, we can guess the antiderivative is $\frac{1}{2}e^{x^2}$.

3. **Check by differentiation**: Differentiate $\frac{1}{2}e^{x^2}$:

$$\frac{d}{dx}\left(\frac{1}{2}e^{x^2}\right) = e^{x^2} \cdot x$$

   This matches the original function.

4. **Include the constant of integration**: The antiderivative is:

$$\int xe^{x^2}\, dx = \frac{1}{2}e^{x^2} + C$$

Key Tips for Inspection Method:

- **Look for patterns**: Recognize derivatives of common functions (polynomials, exponentials, logarithms, trigonometric functions, etc.).
- **Apply the chain rule in reverse**: If the function includes a composition (like $\cos(2x)$ or e^{3x}), think about undoing the chain rule by dividing by the derivative of the inner function.
- **Practice with standard forms**: The more you practice with standard integrals, the better you'll become at quickly identifying patterns.

By following these steps and practicing regularly, you will improve your ability to find antiderivatives by inspection quickly and accurately.