Integrals by Parts

Integration by Parts is a technique used to evaluate integrals where the product of two functions is involved. It is based on the **product rule for differentiation**, which is rearranged for integration:

$$\int u\,dv = uv - \int v\,du$$

Here:

- u: A function you select to differentiate.
- dv: A function you select to integrate.

Steps to Solve Using Integration by Parts

1. Identify u and dv from the given integral $\int u\,dv$.
2. Compute $du = \frac{d}{dx}(u)\,dx$.
3. Compute $v = \int dv$.
4. Substitute into the formula: $\int u\,dv = uv - \int v\,du$.
5. Simplify and solve the remaining integral.

Choosing u and dv

A good rule of thumb is to use the **LIATE** mnemonic:

- **L**: Logarithmic functions ($\ln x$)
- **I**: Inverse trigonometric functions ($\arctan x, \arcsin x$, etc.)
- **A**: Algebraic functions ($x, x^2, \ldots$)
- **T**: Trigonometric functions ($\sin x, \cos x$, etc.)
- **E**: Exponential functions (e^x, a^x)

Choose u as the function appearing earlier in the list.

Examples

Example 1: $\int xe^x\,dx$

1. Let $u = x$ (algebraic) and $dv = e^x\,dx$ (exponential).
2. Compute:
   - $du = dx$
   - $v = \int e^x\,dx = e^x$
3. Substitute into the formula:

$$\int xe^x\,dx = uv - \int v\,du = xe^x - \int e^x\,dx$$

4. Solve the remaining integral:

$$\int xe^x \, dx = xe^x - e^x + C$$

Final Answer:

$$\int xe^x \, dx = e^x(x-1) + C$$

Example 2: $\int \ln x \, dx$

1. Let $u = \ln x$ (logarithmic) and $dv = dx$ (constant).
2. Compute:
   - $du = \frac{1}{x} \, dx$
   - $v = \int dx = x$
3. Substitute into the formula:

$$\int \ln x \, dx = uv - \int v \, du = x \ln x - \int x \frac{1}{x} \, dx$$

4. Solve the remaining integral:

$$\int \ln x \, dx = x \ln x - \int 1 \, dx = x \ln x - x + C$$

Final Answer:

$$\int \ln x \, dx = x(\ln x - 1) + C$$

Example 3: $\int x^2 \sin x \, dx$

1. Let $u = x^2$ (algebraic) and $dv = \sin x \, dx$ (trigonometric).
2. Compute:
   - $du = 2x \, dx$
   - $v = \int \sin x \, dx = -\cos x$
3. Substitute into the formula:

$$\int x^2 \sin x \, dx = uv - \int v \, du = -x^2 \cos x + \int 2x \cos x \, dx$$

4. Use integration by parts again for $\int 2x \cos x \, dx$, letting $u = x$, $dv = \cos x \, dx$:
   - $du = dx$
   - $v = \sin x$

$$\int 2x \cos x \, dx = 2(x \sin x - \int \sin x \, dx) = 2(x \sin x + \cos x)$$

5. Substitute back:

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2(x \sin x + \cos x) + C$$

Final Answer:

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2 \cos x + C$$
