Integrals Substitution Technique Problems

Here is the key (solution outline) for the 10 integral problems based on substitution technique:

1. $\int \sin(2x)\,dx$
   - Substitution: $u = 2x$, so $du = 2dx$.
   - Solution: $\frac{-\cos(2x)}{2} + C$
2. $\int e^{3x}\,dx$
   - Substitution: $u = 3x$, so $du = 3dx$.
   - Solution: $\frac{e^{3x}}{3} + C$
3. $\int \frac{1}{1+x^2}\,dx$
   - Standard integral form: $\tan^{-1}(x) + C$
4. $\int \frac{1}{\sqrt{1-x^2}}\,dx$
   - Standard integral form: $\sin^{-1}(x) + C$
5. $\int \frac{e^x}{1+e^x}\,dx$
   - Substitution: $u = 1 + e^x$, so $du = e^x dx$.
   - Solution: $\ln(1 + e^x) + C$
6. $\int \cos^2(x)\,dx$
   - Use identity: $\cos^2(x) = \frac{1+\cos(2x)}{2}$.
   - Solution: $\frac{x}{2} + \frac{\sin(2x)}{4} + C$
7. $\int \frac{x}{\sqrt{1+x^2}}\,dx$
   - Substitution: $u = 1 + x^2$, so $du = 2xdx$.
   - Solution: $\sqrt{1 + x^2} + C$
8. $\int xe^{x^2}\,dx$
   - Substitution: $u = x^2$, so $du = 2xdx$.
   - Solution: $\frac{e^{x^2}}{2} + C$
9. $\int \frac{dx}{x\ln(x)}$
   - Substitution: $u = \ln(x)$, so $du = \frac{dx}{x}$.
   - Solution: $\ln|\ln(x)| + C$
10. $\int \sin(x)e^{\cos(x)}\,dx$
    - Substitution: $u = \cos(x)$, so $du = -\sin(x)dx$.
    - Solution: $-e^{\cos(x)} + C$

These solutions involve common techniques like basic substitution and the use of standard integrals.