We are given:

- A, B are invertible 3×3 matrices
- $\det(ABA') = 8$
- $\det(AB^{-1}) = 8$
- Find: $\det(BA^{-1}B')$

Step 1: Use determinant properties

Key Properties:

1. $\det(XY) = \det(X)\det(Y)$
2. $\det(X^{-1}) = \frac{1}{\det(X)}$
3. $\det(X') = \det(X)$ (since transpose doesn't change determinant)

Step 2: Use first condition

$$\det(ABA') = \det(A)\det(B)\det(A') = \det(A)\det(B)\det(A) = \det(B)(\det(A))^2 = 8 \quad (1)$$

Step 3: Use second condition

$$\det(AB^{-1}) = \det(A)\det(B^{-1}) = \det(A) \cdot \frac{1}{\det(B)} = 8 \quad (2)$$

Step 4: Multiply Equations (1) and (2)

Multiply (1) and (2):

$$\left[\det(B)(\det(A))^2\right] \cdot \left[\frac{\det(A)}{\det(B)}\right] = 8 \cdot 8$$

Simplify:

$$\det(A)^3 = 64 \Rightarrow \det(A) = 4$$

Plug back into (2):

$$4 \cdot \frac{1}{\det(B)} = 8 \Rightarrow \det(B) = \frac{1}{2}$$

Step 5: Compute $\det(BA^{-1}B')$

$$\det(BA^{-1}B') = \det(B) \cdot \det(A^{-1}) \cdot \det(B') = \det(B) \cdot \frac{1}{\det(A)} \cdot \det(B) = \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{16}$$

Final Answer:

$$\boxed{\frac{1}{16}} \quad \text{(Option d)}$$

Is this conversation helpful so far?