We are given a system of equations:

1. $x + (\sin\alpha)y = 1$
2. $(\sin\alpha)x + 4y = 2$

With the condition:

- $x \geq \frac{4}{5}$
- $y \leq \frac{1}{2}$

We are to find the range of α satisfying the system and the constraints.

Step 1: Solve the system of equations

Label the equations:

(1) $x + (\sin\alpha)y = 1$

(2) $(\sin\alpha)x + 4y = 2$

Use equation (1) to express x in terms of y:

$$x = 1 - (\sin\alpha)y$$

Substitute into (2):

$$(\sin\alpha)(1 - (\sin\alpha)y) + 4y = 2$$

$$\sin\alpha - (\sin^2\alpha)y + 4y = 2$$

Group y terms:

$$\sin\alpha + y(4 - \sin^2\alpha) = 2$$

Solve for y:

$$y = \frac{2 - \sin\alpha}{4 - \sin^2\alpha}$$

Substitute into $x = 1 - (\sin\alpha)y$:

$$x = 1 - (\sin\alpha)\left(\frac{2 - \sin\alpha}{4 - \sin^2\alpha}\right)$$

Step 2: Apply constraints

We are given:

$$x \geq \frac{4}{5}, \quad y \leq \frac{1}{2}$$

We analyze when both these inequalities are satisfied.

Let $s = \sin\alpha$, where $s \in (0, 1)$

From earlier:

$$y = \frac{2-s}{4-s^2}, \quad x = 1 - s \cdot y = 1 - s \cdot \left(\frac{2-s}{4-s^2}\right)$$

Now we test these expressions numerically to find a range of $s = \sin\alpha$ for which:

$$x \geq \frac{4}{5}, \quad y \leq \frac{1}{2}$$

Try $s = \frac{1}{2}$ (i.e. $\alpha = \frac{\pi}{6}$):

- $y = \frac{2-1/2}{4-(1/2)^2} = \frac{3/2}{4-1/4} = \frac{3/2}{15/4} = \frac{3/2 \cdot 4}{15} = \frac{6}{15} = 0.4 \leq 0.5$ ✅
- $x = 1 - (1/2)(0.4) = 1 - 0.2 = 0.8 = \frac{4}{5}$ ✅

Try slightly more, $s = 0.6$:

- $y = \frac{2-0.6}{4-0.36} = \frac{1.4}{3.64} \approx 0.3846$
- $x = 1 - 0.6(0.3846) \approx 1 - 0.2308 = 0.7692 < 0.8$ ❌

Try slightly less, $s = 0.4$:

- $y = \frac{2-0.4}{4-0.16} = \frac{1.6}{3.84} \approx 0.4167$
- $x = 1 - 0.4(0.4167) \approx 1 - 0.1667 = 0.8333 > 0.8$ ✅

So the values of $\sin\alpha$ for which both conditions hold lie between 0.4 and 0.5 approximately.

This corresponds to:

- $\alpha \in [\arcsin(0.4), \arcsin(0.5)]$
- $\alpha \in [\arcsin(0.4), \frac{\pi}{6}]$
- $\arcsin(0.4) \approx 0.4115 \text{ rad}, \quad \frac{\pi}{6} \approx 0.5236 \text{ rad}$

This matches option **(d)**:

$$\boxed{\alpha \in \left[0, \frac{\pi}{6}\right]}$$