Solution

DETERMINANTS

Class 12 - Mathematics

1.
(d) $\alpha \in \left|0, \frac{\pi}{6}\right|$
Explanation:
$\alpha \in \left|0, \frac{\pi}{6}\right|$

2.
(c) 1 , 2 and 3
Explanation:
Expanding along C_1
$\begin{vmatrix} 1-x & 2 & 3 \\ 0 & 2-x & 0 \\ 0 & 2 & 3-x \end{vmatrix} = 0 \Rightarrow (1 - x)(2 - x)(3 - x) = 0 \Rightarrow x = 1, 2, 3.$

3.
(d) $\begin{bmatrix} 2 & 0 & 0 \\ -5 & 1 & 0 \\ 32 & -6 & 2 \end{bmatrix}$
Explanation:
adj. A = $\begin{bmatrix} 2 & -5 & 32 \\ 0 & 1 & -6 \\ 0 & 0 & 2 \end{bmatrix}' = \begin{bmatrix} 2 & 0 & 0 \\ -5 & 1 & 0 \\ 32 & -6 & 2 \end{bmatrix}$

4. **(a)** 25
Explanation:
$|A| = 5$, $|\text{adj } A| = |A|^{3-1} = |A|^2 = 5^2 = 25$

5.
(c) $a_{11} A_{11} + a_{21} A_{21} + a_{31} A_{31}$
Explanation:
$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$
Expanding along Column 1
$\Delta = (-1)^{1+1} \times a_{11} \times \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{2+1} \times a_{21} \times \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{3+1} \times a_{31} \times \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$
$\Delta = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

6.
(d) x = 1, y = 1
Explanation:
$\begin{pmatrix} x & y \\ 3y & x \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$
$= \begin{pmatrix} x \times 1 + y \times 2 \\ 3y \times 1 + x \times 2 \end{pmatrix}$
$= \begin{pmatrix} x + 2y \\ 3y + 2x \end{pmatrix}$
Comparing with R.H.S
x + 2y = 3 ...(i)
2x + 3y = 5 ...(ii)

(i) x 2 - (ii)

2x + 4y - 2x + 3y = 6 - 5

y = 1

Putting y in (i)

x + 2(1) = 3

x = 1

7.

(c) |A| + |A'| $\neq$ 0

Explanation:

Because, the determinant of a matrix and its transpose are always equal that is |A| = |A'|

8.

(d) 27

Explanation:

Since the matrix is of order 3 so 3 will be taken common from each row or column.

So, k = 27

9.

(d) $\frac{1}{16}$

Explanation:

$\frac{1}{16}$

10.

(b) $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

Explanation:

A = $\begin{vmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{vmatrix}$

A^TA = $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}\begin{vmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{vmatrix}$

= $\begin{vmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha\sin\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{vmatrix}$

= $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

11. Now,L.H.S. = $(AB)^{-1}$ $\Rightarrow$ AB = $= \begin{bmatrix} 9 & -1 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} -4 & 3 \\ 5 & -4 \end{bmatrix} = \begin{bmatrix} -36-5 & 27+4 \\ -24-10 & 18+8 \end{bmatrix} = \begin{bmatrix} -41 & 31 \\ -34 & 26 \end{bmatrix}$

adj AB = $\begin{bmatrix} C_{11} & C_{33} \\ C_{21} & C_{22} \end{bmatrix}$

Cofactor of $C_{11} = (-1)^{1+1}$ (26) = 26; $C_{12} = (-1)^{1+2}$ (-34) = 34

$C_{21} = (-1)^{2+1}$ (31) = -31,$C_{22} = (-1)^{2+2}$ (-41) = -41

$\therefore \quad \text{adj}\, AB = \begin{bmatrix} 26 & 34 \\ -31 & -41 \end{bmatrix} \quad \Rightarrow \text{ adj } AB = \begin{bmatrix} 26 & -31 \\ 34 & -41 \end{bmatrix}$

|AB| = $\begin{vmatrix} -41 & 31 \\ -34 & 26 \end{vmatrix}$ = (1066 - 1054) = 12

$\therefore$ $(AB)^{-1}$ $\frac{1}{|AB|}(\text{adj}\, AB) = \frac{1}{12}\begin{bmatrix} 26 & -31 \\ 34 & -41 \end{bmatrix}$

R.H.S. = B^{-1} A^{-1} $\Rightarrow B = \begin{bmatrix} -4 & 3 \\ 5 & -4 \end{bmatrix} \Rightarrow \text{adj}\, B = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

Cofactor of $C_{11} = (-1)^{1+1}$ (-4) = -4; $C_{12} = (-1)^{1+2}$ (5) = -5

$C_{21} = (-1)^{2+1}(3) = -3, C_{22} = (-1)^{2+2}(-4) = -4$

$\text{adj}\, B = \begin{bmatrix} -4 & -5 \\ -3 & -4 \end{bmatrix} \Rightarrow \text{adj}\, B = \begin{bmatrix} -4 & -3 \\ -5 & -4 \end{bmatrix} \Rightarrow |B| = \begin{vmatrix} -4 & 3 \\ 5 & -4 \end{vmatrix}$ = (16 - 15) = 1

$\therefore B^{-1} = \frac{1}{|B|}(\text{adj}\, B) = \frac{1}{1}\begin{bmatrix} -4 & -3 \\ -5 & -4 \end{bmatrix}$

$A = \begin{bmatrix} 9 & -1 \\ 6 & -2 \end{bmatrix} \Rightarrow \text{adj}\, A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

Cofactor of $C_{11} = (-1)^{1+1}(-2) = -4$; $C_{12} = (-1)^{1+2}(6) = -6$

$C_{21} = (-1)^{2+1}(-1) = 1, C_{22} = (-1)^{2+2}(9) = 9$

$|A| = \begin{vmatrix} 9 & -1 \\ 6 & -2 \end{vmatrix}$ = (-18 + 6) = -12

$\therefore A^{-1} = \frac{1}{|A|}(\text{adj}\, A) = \frac{1}{12}\begin{bmatrix} -2 & 1 \\ -6 & 9 \end{bmatrix}$

Now,we have to find $B^{-1} A^{-1} = \frac{1}{12}\begin{bmatrix} -4 & -3 \\ -5 & -4 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ -6 & 9 \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 8+18 & -4-27 \\ 10+24 & -5-36 \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 26 & -31 \\ 34 & -41 \end{bmatrix}$

Therefore, $(AB)^{-1} = B^{-1} \cdot A^{1}$.

12. $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$

$= \frac{1}{2}[3(2-1) - 8(-4-5) + 1(-4-10)]$

$= \frac{1}{2}[3 + 72 - 14] = \frac{61}{2}$

13. Let the numbers are x, y, z

3x + 5y – 4z = 6000 …(i)

Also,

2x – 3y + z = 5000 …(ii)

Again,

– x + 4y + 6z = 13000 …(iii)

$$\begin{bmatrix} 3 & 5 & -4 \\ 2 & -3 & 1 \\ -1 & 4 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6000 \\ 5000 \\ 13000 \end{bmatrix}$$

A X = B

|A| = 3(– 18 – 4) – 2(30 + 16) – 1(5 – 12)

= 3(– 22) – 2(46) + 7

= – 66 – 92 + 7

= – 151

Hence, the unique solution given by $x = A^{-1}B$

$C_{11} = (-1)^{1+1}(-18-4) = -22$

$C_{12} = (-1)^{1+2}(12+1) = -13$

$C_{13} = (-1)^{1+3}(8-3) = 5$

$C_{21} = (-1)^{2+1}(30+16) = -46$

$C_{22} = (-1)^{2+2}(18-4) = 14$

$C_{23} = (-1)^{2+3}(12+5) = -17$

$C_{31} = (-1)^{3+1}(5-12) = -7$

$C_{32} = (-1)^{3+2}(3+8) = -11$

$C_{33} = (-1)^{3+3}(-9-10) = -19$

$$\text{Adj A} = \begin{bmatrix} -22 & -46 & -7 \\ -13 & 14 & -11 \\ 5 & -17 & -19 \end{bmatrix}$$

$$x = A^{-1}B = \frac{1}{|A|}(\text{adj}\, A)B$$

$$X = \frac{1}{6}\begin{bmatrix} -22 & 46 & -7 \\ -13 & 14 & -11 \\ 5 & -17 & -19 \end{bmatrix}\begin{bmatrix} 6000 \\ 5000 \\ 13000 \end{bmatrix}$$

$$X = \frac{1}{-151}\begin{bmatrix} -132000 - 23000 - 91000 \\ -78000 + 70000 - 143000 \\ -3000 - 85000 - 247000 \end{bmatrix}$$

$$\begin{bmatrix} \text{x} \\ \text{y} \\ \text{z} \end{bmatrix} = \begin{bmatrix} 3000 \\ 1000 \\ 2000 \end{bmatrix}$$

Hence, x = 3000, y = 1000 and z = 2000

14. We are given that,

A = non-singular matrix

B = non-singular matrix

Order of A = Order of B

We need to find whether AB is singular or non-singular

Let us recall the definition of non-singular matrix.

Non-singular matrix, also called regular matrix, is a square matrix that is not singular, i.e., one that has a matrix inverse.

We can say that, a square matrix A is non-singular matrix if its determinant is non-zero, i.e., |A| $\neq$ 0.

While a singular matrix is a square matrix that doesn't have a matrix inverse. Also, the determinant is zero, i.e., |A| = 0...

So,

By definition, |A| $\neq$ 0 and |B| $\neq$ 0 since A and B are non-singular matrices.

Let,

Order of A = Order of B = n $\times$ n

$\Rightarrow$ Matrices A and B can be multiplied

$\Rightarrow$ A $\times$ B = AB

If we have matrices A and B of same order then we can say that,

|AB| = 0 if either |A| or |B| = 0.

And it is clear that, |A|, |B| $\neq$ 0.

$\Rightarrow$ |AB| $\neq$ 0

And if |AB| $\neq$ 0, then by definition AB is s non-singular matrix.

Thus, AB is a singular matrix

15. $AB = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 5 \\ 5 & -14 \end{bmatrix}$$

$$|AB| = -11 \neq 0$$

$$(AB)^{-1} = \frac{1}{11} adj\,(AB)$$

$$= \frac{-1}{11}\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$$

$$|A| = -11 \neq 0,\ |B| = 1 \neq 0$$

$$A^{-1} = \frac{-1}{1}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{1}{1}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{-1}{11}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$$

$$= \frac{-1}{11}\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix}$$

$= \frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$

Hence prove.

16. Given equations are: -

x + y = 5

y + z = 3

x + z = 4

It can be written as $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix}$

So by comparing with theorem, lets find D , D_1 and D_2

$\Rightarrow$ D = $\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$

Solving determinant, expanding along R_1

⇒ D = 1[1] - 1[- 1] + 0[- 1]

⇒ D = 1 + 1 + 0

⇒ D = 2

⇒ D = 2

Again, Solve D_1 formed by replacing 1st column by B matrices

$\Rightarrow$ D_1 = $\begin{vmatrix} 5 & 1 & 0 \\ 3 & 1 & 1 \\ 4 & 0 & 1 \end{vmatrix}$

Solving determinant, expanding along R_1

$\Rightarrow$ D_1 = 5[1] - 1[(3)(1) - (4)(1)] + 0

$\Rightarrow$ D_1 = 5 - 1[3 - 4] + 0

$\Rightarrow$ D_1 = 5 + 1 + 0

$\Rightarrow$ D_1 = 6

Again, Solve D_2 formed by replacing 2nd column by B matrices

$\Rightarrow$ D_2 = $\begin{vmatrix} 1 & 5 & 0 \\ 0 & 3 & 1 \\ 1 & 4 & 1 \end{vmatrix}$

Expand by R_1

$\Rightarrow$ D_2 = 1[3 - 4] - 5[- 1] + 0

$\Rightarrow$ D_2 = 1[- 1] + 5 + 0

$\Rightarrow$ D_2 = 4

And, Solve D_3 formed by replacing 3rdcolumn by B matrices

$\Rightarrow$ D_3 = $\begin{vmatrix} 1 & 1 & 5 \\ 0 & 1 & 3 \\ 1 & 0 & 4 \end{vmatrix}$

Expand by R_1

$\Rightarrow$ D_3 = 1[4 - 0] - 1[0 - 3] + 5[0 - 1]

$\Rightarrow$ D_3 = 1[4] - 1(- 3) + 5(- 1)

$\Rightarrow$ D_3 = 4 + 3 - 5

$\Rightarrow$ D_3 = 2

Thus by Cramer's Rule, we have

$\Rightarrow$ x = $\frac{D_1}{D}$, y = $\frac{D_2}{D}$ and z $= \frac{\mathrm{D}_3}{\mathrm{D}}$

$\Rightarrow$ x = $\frac{6}{2}$, y = $\frac{4}{2}$ and z $= \frac{2}{2}$

x = 3, y= 2 and z =1

17. Given: - Equations are: -

x + y = 1........(i)

x + z = - 6........(ii)

x - y - 2z = 3......(iii)

So by comparing with theorem, lets find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & -2 \end{vmatrix}$$

Solving determinant, expanding along 1st Row we have.

$\Rightarrow$ D = 1[(0)(- 2) - (1)(- 1)] - 1[(- 2)(1) - 1] + 0[- 1 - 0]

$\Rightarrow$ D = 1[0 + 1] - 1[- 3] - 0[- 2]

$\Rightarrow$ D = 1 + 3 + 0

$\Rightarrow$ D = 4

Again, Solve for D_1 formed by replacing 1st column by B matrices

Here

$$B = \begin{vmatrix} 1 \\ -6 \\ 3 \end{vmatrix} \text{ Therefore}$$

$$\Rightarrow D1 = \begin{vmatrix} 1 & 1 & 0 \\ -6 & 0 & 1 \\ 3 & -1 & -2 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$\Rightarrow D_1$ = 1[(0)(- 2) - (1)(- 1)] - 1[(- 2)(- 6) - 3] + 0[6 - 0]

$\Rightarrow D_1$ = 1[0 + 1] - 1[12 - 3] + 0[6]

$\Rightarrow D_1$ = 1[1] - 9 + 0

$\Rightarrow D_1$ = 1 - 9 + 0

$\Rightarrow D_1$ = - 8

Again, Solve for D_2 formed by replacing 2nd column by B matrices

Here

$$B = \begin{vmatrix} 1 \\ -6 \\ 3 \end{vmatrix} \text{ therefore}$$

$$\Rightarrow D_2 = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -6 & 1 \\ 1 & 3 & -2 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$\Rightarrow D_2$ = 1[(- 6)(- 2) - (1)(3)] - 1[(- 2)(1) - 1] + 0[3 + 6]

$\Rightarrow D_2$ = 1[12 - 3] - 1(- 2 - 1) + 0(9)

$\Rightarrow D_2$ = 9 + 3

$\Rightarrow D_2$ = 12

And, Solve D_3 formed by replacing 3rd column by B matrices

Here

$$B = \begin{vmatrix} 1 \\ -6 \\ 3 \end{vmatrix} \text{ Therefore}$$

$$\Rightarrow D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & -6 \\ 1 & -1 & 3 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$\Rightarrow D_3$ = 1[(0)(3) - (- 1)(- 6)] - 1[(3)(1) - 1(- 6)] + 1[- 1 + 0]

$\Rightarrow D_3 = 1[0 - 6] - 1(3 + 6) + 1(-1)$

$\Rightarrow D_3 = -6 - 9 - 1$

$\Rightarrow D_3 = -16$

Thus by Cramer's Rule, we have

$\Rightarrow \mathrm{x} = \frac{\mathrm{D}_1}{\mathrm{D}}$

$\Rightarrow \mathrm{x} = \frac{-8}{4}$

$\Rightarrow x = -2$

again,

$\Rightarrow \mathrm{y} = \frac{\mathrm{D}_2}{\mathrm{D}}$

$\Rightarrow y = \frac{12}{4}$

$\Rightarrow y = 3$

and

$\Rightarrow \mathrm{z} = \frac{\mathrm{D}_3}{\mathrm{D}}$

$\Rightarrow \mathrm{z} = \frac{-16}{4}$

$\Rightarrow z = -4$

18. Let the three numbers be x,y and z.Then,

x + y + z = 6

y + 3z = 11

x + z = 2y

This system can be written as AX = B whose

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$|A| = 9 \neq 0$

$A_{11} = 7, A_{12} = 3, A_{13} = -1$

$A_{21} = -3, A_{22} = 0, A_{23} = 3$

$A_{31} = 2, A_{32} = -3, A_{33} = 1$

$$adjA = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

x=1, y=2, z=3

19. Three equations are formed from the given statements:

3x + 2y + z = 2200

4x + y + 3z = 3100 and

x + y + z = 1200

Converting the system of equations in matrix form we get,

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

i.e. AX = B

where A = $\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$

$\Rightarrow X = A^{-1} B$

$A^{-1} = \frac{1}{|A|}$ (Adj A)

$|A| = 3(1 - 3) - 2(4 - 3) + 1(4 - 1) = -6 - 2 + 3 = -5$

$$\text{Adj } A = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{-5} \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{5} \begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix} \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 4400 + 3100 - 6000 \\ 2200 - 6200 + 6000 \\ -6600 + 3100 + 6000 \end{bmatrix}$$

$$= \begin{bmatrix} 300 \\ 400 \\ 500 \end{bmatrix}$$

$\Rightarrow x = 300, y = 400$ and $z = 500$

i.e. The award money for each value are Rs.300 for Tolerance, Rs.400 for Kindness and Rs.500 for Leadership.