

Calculus Basics Class 12



Lecture notes for continuity and differentiability class 12



Continuity and Differentiability - Class 12

1. Continuity

- **Definition:** A function $f(x)$ is continuous at a point $x = c$ if:

$$\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = f(c)$$

- **Types of Discontinuities:**

- **Removable:** $\lim_{x \rightarrow c} f(x)$ exists, but $\lim_{x \rightarrow c} f(x) \neq f(c)$.
- **Jump:** $\lim_{x \rightarrow c^-} f(x) \neq \lim_{x \rightarrow c^+} f(x)$.
- **Infinite:** $\lim_{x \rightarrow c} f(x)$ is $\pm\infty$.

2. Differentiability

- **Definition:** A function $f(x)$ is differentiable at a point $x = c$ if the derivative $f'(c)$ exists:

$$f'(c) = \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}$$

- **Differentiability implies Continuity:** If $f(x)$ is differentiable at $x = c$, then it is continuous at $x = c$. The converse is not necessarily true.

3. Derivatives of Elementary Functions

- **Power Rule:** $\frac{d}{dx}(x^n) = nx^{n-1}$
- **Exponential Functions:** $\frac{d}{dx}(e^x) = e^x$, $\frac{d}{dx}(a^x) = a^x \ln(a)$
- **Logarithmic Functions:** $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$
- **Trigonometric Functions:**
 - $\frac{d}{dx}(\sin(x)) = \cos(x)$
 - $\frac{d}{dx}(\cos(x)) = -\sin(x)$
 - $\frac{d}{dx}(\tan(x)) = \sec^2(x)$
- **Inverse Trigonometric Functions:**
 - $\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$
 - $\frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}$
 - $\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$

4. Rules of Differentiation

- **Sum Rule:** $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
- **Product Rule:** $\frac{d}{dx}[f(x) \cdot g(x)] = f(x)g'(x) + f'(x)g(x)$
- **Quotient Rule:** $\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$
- **Chain Rule:** If $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

5. Higher Order Derivatives

- **Second Derivative:** The derivative of the first derivative, denoted as $f''(x)$ or $\frac{d^2y}{dx^2}$.

6. Applications of Derivatives

- **Rate of Change:** Derivative represents the rate of change of a function.
- **Tangent and Normal:**
 - Slope of the tangent at $x = c$ is $f'(c)$.
 - Equation of tangent: $y - f(c) = f'(c)(x - c)$.
 - Slope of the normal is $-\frac{1}{f'(c)}$.
 - Equation of normal: $y - f(c) = -\frac{1}{f'(c)}(x - c)$.

7. Rolle's Theorem

- If $f(x)$ is continuous on $[a, b]$, differentiable on (a, b) , and $f(a) = f(b)$, then there exists at least one $c \in (a, b)$ such that $f'(c) = 0$.

8. Mean Value Theorem

- If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) , then there exists at least one $c \in (a, b)$ such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$