

Chapter 5: Continuity and Differentiability

5.1 Introduction

- Overview of differentiation from previous classes
- Introduction to continuity, differentiability, and their relationship
- Differentiation of inverse trigonometric functions
- Introduction to exponential and logarithmic functions
- Fundamental theorems

5.2 Continuity

- Informal examples of continuity
- Mathematical definition of continuity
- Examples of continuous and discontinuous functions
- Algebra of continuous functions
 - Sum, difference, product, and quotient of continuous functions

5.2.1 Algebra of Continuous Functions

- Theorem: Continuity of sum, difference, product, and quotient
- Remarks and special cases
- Examples proving the continuity of rational and trigonometric functions

5.3 Differentiability

- Definition of the derivative
- Rules of differentiation
 - Power rule, product rule, quotient rule
- Relationship between differentiability and continuity
- Differentiability implies continuity
- Counterexamples: Continuous but not differentiable functions

5.3.1 Derivatives of Composite Functions

- Chain rule
- Examples of applying the chain rule
- Extended chain rule for compositions of multiple functions

5.3.2 Derivatives of Implicit Functions

- Definition and examples

- Implicit differentiation techniques
- Examples of implicit differentiation

5.3.3 Derivatives of Inverse Trigonometric Functions

- Differentiation of inverse trigonometric functions
- Examples and standard results
- Table of derivatives of inverse trigonometric functions

5.4 Exponential and Logarithmic Functions

- Introduction to exponential functions
- Properties of exponential functions
- Introduction to logarithmic functions
- Properties of logarithmic functions
- Change of base rule
- Properties of logarithms

5.5 Logarithmic Differentiation

- Technique of logarithmic differentiation
- Examples and applications
- Differentiation of functions of the form $[u(x)]^{v(x)}$

Exercises

- A variety of exercises covering all topics discussed in the chapter

Example Problems and Solutions

- Detailed solutions to examples of continuous and differentiable functions
- Applications of differentiation rules
- Implicit differentiation examples
- Differentiation of inverse trigonometric functions
- Logarithmic differentiation examples