



## Example 6: Problem Statement and Solution

### Problem Statement:

The volume of a spherical balloon is increasing at the rate of  $20 \text{ cm}^3/\text{sec}$ . Find the rate of change of its surface area at an instant when the radius is 5 cm.

### Solution:

Let  $r$  be the radius,  $v$  be the volume, and  $s$  be the surface area of the spherical balloon at any time  $t$ .

- Given:
  - $\frac{dv}{dt} = 20 \text{ cm}^3/\text{sec}$ .
  - $r = 5 \text{ cm}$ .

The volume  $v$  of a sphere is given by:

$$v = \frac{4}{3}\pi r^3$$

The surface area  $s$  of a sphere is given by:

$$s = 4\pi r^2$$

### Step 1: Differentiate Volume with Respect to Time

$$\frac{dv}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$$

$$20 = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{20}{4\pi r^2} = \frac{5}{\pi r^2}$$

### Step 2: Differentiate Surface Area with Respect to Time

$$\frac{ds}{dt} = 4\pi \cdot 2r \cdot \frac{dr}{dt}$$

$$\frac{ds}{dt} = 8\pi r \cdot \frac{5}{\pi r^2} = \frac{40}{r}$$

**Step 3: Substitute  $r = 5$  cm**

$$\frac{ds}{dt} = \frac{40}{5} = 8 \text{ cm}^2/\text{sec}$$

Thus, the rate of change of surface area is  $8 \text{ cm}^2/\text{sec}$ .

## Similar Problems

### 1. Problem 1:

- The volume of a spherical balloon is increasing at the rate of  $30 \text{ cm}^3/\text{sec}$ . Find the rate of change of its surface area at an instant when the radius is 6 cm.

### 2. Problem 2:

- The volume of a spherical balloon is increasing at the rate of  $25 \text{ cm}^3/\text{sec}$ . Find the rate of change of its surface area at an instant when the radius is 4 cm.

### 3. Problem 3:

- The volume of a spherical balloon is increasing at the rate of  $15 \text{ cm}^3/\text{sec}$ . Find the rate of change of its surface area at an instant when the radius is 7 cm.

### 4. Problem 4:

- The volume of a spherical balloon is increasing at the rate of  $10 \text{ cm}^3/\text{sec}$ . Find the rate of change of its surface area at an instant when the radius is 8 cm.

### 5. Problem 5:

- The volume of a spherical balloon is increasing at the rate of  $35 \text{ cm}^3/\text{sec}$ . Find the rate of change of its surface area at an instant when the radius is 3 cm.

### 6. Problem 6:

- The volume of a spherical balloon is increasing at the rate of  $50 \text{ cm}^3/\text{sec}$ . Find the rate of change of its surface area at an instant when the radius is 10 cm.