Tangent Normal Equations Examples

Tangent and Normal in Derivatives - Lecture Notes with Examples

Tangent to a Curve

The tangent to a curve at a given point is a straight line that just "touches" the curve at that point. The slope of this tangent line at the point (x_0, y_0) is given by the derivative of the function $y = f(x)$ at $x = x_0$.

Equation of the Tangent Line:
If $y = f(x)$ and the slope at $x = x_0$ is $f'(x_0)$, the equation of the tangent line at $(x_0, f(x_0))$ is:
$y - f(x_0) = f'(x_0)(x - x_0)$

Example 1:
Find the equation of the tangent line to the curve $y = x^2$ at the point $(1, 1)$.

Solution:

1. Compute the derivative: $f'(x) = 2x$
2. Evaluate the derivative at $x = 1$: $f'(1) = 2$
3. Use the point $(1, 1)$ in the tangent line equation:
   $y - 1 = 2(x - 1)$
   $y = 2x - 1$

Normal to a Curve

The normal to a curve at a given point is a straight line perpendicular to the tangent at that point. The slope of the normal line is the negative reciprocal of the slope of the tangent line.

Equation of the Normal Line:
If $y = f(x)$ and the slope of the tangent at $x = x_0$ is $f'(x_0)$, the slope of the normal line is $-\frac{1}{f(x_0)}$. The equation of the normal line at $(x_0, f(x_0))$ is:
$y - f(x_0) = -\frac{1}{f(x_0)}(x - x_0)$

Example 2:
Find the equation of the normal line to the curve $y = x^2$ at the point $(1, 1)$.

Solution:

1. Compute the derivative: $f'(x) = 2x$
2. Evaluate the derivative at $x = 1$: $f'(1) = 2$
3. Compute the slope of the normal line: $-\frac{1}{2}$
4. Use the point $(1, 1)$ in the normal line equation:
   $y - 1 = -\frac{1}{2}(x - 1)$

$y - 1 = -\frac{1}{2}x + \frac{1}{2}$
$y = -\frac{1}{2}x + \frac{3}{2}$

Additional Examples and Exercises

Example 3:
Find the equation of the tangent and normal lines to the curve $y = \sin(x)$ at $x = \frac{\pi}{4}$.

Solution:

1. Compute the derivative: $f'(x) = \cos(x)$
2. Evaluate the derivative at $x = \frac{\pi}{4}$: $f'\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$
3. Tangent line equation at $\left(\frac{\pi}{4}, \sin\left(\frac{\pi}{4}\right)\right) = \left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$:
   $y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$
   $y = \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}\pi}{8} + \frac{\sqrt{2}}{2}$
4. Normal line equation with slope $-\frac{2}{\sqrt{2}} = -\sqrt{2}$:
   $y - \frac{\sqrt{2}}{2} = -\sqrt{2}\left(x - \frac{\pi}{4}\right)$
   $y = -\sqrt{2}x + \frac{\sqrt{2}\pi}{4} + \frac{\sqrt{2}}{2}$

Practice Problems:

1. Find the equation of the tangent and normal lines to the curve $y = \ln(x)$ at $x = 1$.
2. Determine the tangent and normal lines to the curve $y = e^x$ at $x = 0$.
3. Compute the tangent and normal lines for the curve $y = x^3 - 3x$ at $x = 1$.