Tangent Normal Equations Examples

Practice Problems on Tangent and Normal Lines to the Curve in Derivatives

Problem 1:

Find the equation of the tangent and normal lines to the curve $y = x^3 - 3x^2 + 2x$ at the point $x = 2$.

Problem 2:

Determine the equation of the tangent and normal lines to the curve $y = \ln(x)$ at the point $x = e$.

Problem 3:

Find the tangent and normal lines to the curve $y = \sin(x)$ at the point $x = \frac{\pi}{6}$.

Problem 4:

Compute the equations of the tangent and normal lines to the curve $y = e^x$ at the point $x = 1$.

Problem 5:

Find the equation of the tangent and normal lines to the curve $y = \sqrt{x}$ at the point $x = 4$.

Problem 6:

Determine the equations of the tangent and normal lines to the curve $y = \frac{1}{x}$ at the point $x = 1$.

Solutions to Practice Problems on Tangent and Normal Lines to the Curve in Derivatives

Solution to Problem 1:

Given $y = x^3 - 3x^2 + 2x$ at $x = 2$:

1. Compute the derivative: $y' = 3x^2 - 6x + 2$.
2. Evaluate the derivative at $x = 2$: $y'(2) = 3(2)^2 - 6(2) + 2 = 12 - 12 + 2 = 2$.
3. Tangent line equation: $y - y(2) = y'(2)(x - 2)$.
4. Normal line equation: $y - y(2) = -\frac{1}{y'(2)}(x - 2)$.

Solution to Problem 2:

Given $y = \ln(x)$ at $x = e$:

1. Compute the derivative: $y' = \frac{1}{x}$.

2. Evaluate the derivative at $x = e$: $y'(e) = \frac{1}{e}$.
3. Tangent line equation: $y - \ln(e) = \frac{1}{e}(x - e)$.
4. Normal line equation: $y - \ln(e) = -e(x - e)$.

Solution to Problem 3:

Given $y = \sin(x)$ at $x = \frac{\pi}{6}$:

1. Compute the derivative: $y' = \cos(x)$.
2. Evaluate the derivative at $x = \frac{\pi}{6}$: $y'(\frac{\pi}{6}) = \cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$.
3. Tangent line equation: $y - \sin(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}(x - \frac{\pi}{6})$.
4. Normal line equation: $y - \sin(\frac{\pi}{6}) = -\frac{2}{\sqrt{3}}(x - \frac{\pi}{6})$.

Solution to Problem 4:

Given $y = e^x$ at $x = 1$:

1. Compute the derivative: $y' = e^x$.
2. Evaluate the derivative at $x = 1$: $y'(1) = e$.
3. Tangent line equation: $y - e = e(x - 1)$.
4. Normal line equation: $y - e = -\frac{1}{e}(x - 1)$.

Solution to Problem 5:

Given $y = \sqrt{x}$ at $x = 4$:

1. Compute the derivative: $y' = \frac{1}{2\sqrt{x}}$.
2. Evaluate the derivative at $x = 4$: $y'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$.
3. Tangent line equation: $y - \sqrt{4} = \frac{1}{4}(x - 4)$.
4. Normal line equation: $y - \sqrt{4} = -4(x - 4)$.

Solution to Problem 6:

Given $y = \frac{1}{x}$ at $x = 1$:

1. Compute the derivative: $y' = -\frac{1}{x^2}$.
2. Evaluate the derivative at $x = 1$: $y'(1) = -1$.
3. Tangent line equation: $y - 1 = -1(x - 1)$.
4. Normal line equation: $y - 1 = 1(x - 1)$.