



Summary of Chapter 7: Applications of Derivatives

This chapter on "Applications of Derivatives" introduces and explores the practical applications of derivatives in calculus, focusing on concepts like rates of change, slopes, optimization, and approximations. Here is a structured summary along with essential formulas:

1. Rate of Change and Slope

- **Definition:** The derivative of a function $f(x)$ represents the instantaneous rate of change of y with respect to x .
- **Formula:** If $f(x)$ is a function, its rate of change or slope at any point is $f'(x)$.

2. Applications in Motion

- **Velocity:** The derivative of position $s(t)$ with respect to time t , i.e., $v(t) = s'(t)$.
- **Acceleration:** The derivative of velocity, $a(t) = v'(t) = s''(t)$.

3. Increasing and Decreasing Functions

- **Monotonicity:** A function is increasing on an interval if $f'(x) > 0$ and decreasing if $f'(x) < 0$.
- **Critical Points:** Points where $f'(x) = 0$ or $f'(x)$ does not exist.

4. Optimization: Maximum and Minimum Values

- **First Derivative Test:**
 - Local maximum: $f'(x)$ changes from positive to negative at a point.
 - Local minimum: $f'(x)$ changes from negative to positive at a point.
- **Second Derivative Test:**
 - If $f''(x) > 0$, the function is concave up (local minimum).
 - If $f''(x) < 0$, the function is concave down (local maximum).
- **Absolute Maxima and Minima:** Analyzed on closed intervals by evaluating the function at critical points and endpoints.

5. Mean Value Theorem (MVT)

- **Statement:** If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) , then there exists at least one $c \in (a, b)$ such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

6. Tangent and Normal Lines

- **Tangent Line:** The equation of the tangent line to $f(x)$ at $x = a$ is:

$$y = f(a) + f'(a)(x - a)$$

- **Normal Line:** The equation of the normal line at $x = a$ is:

$$y = f(a) - \frac{1}{f'(a)}(x - a)$$

7. L'Hôpital's Rule for Indeterminate Forms

- Used to find limits of indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$.
- **Formula:**

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} \quad (\text{if the limit exists})$$

8. Optimization in Real-Life Applications

- **Geometric Optimization:** Problems involving maximizing areas, minimizing costs, etc., using derivatives to find optimal values.
- **Formulas:**
 - Area or volume formulas vary based on the shape and constraints, e.g., optimizing a rectangle's area with a fixed perimeter involves setting up and solving $f(x)$ in terms of the perimeter constraint.

Key Formulas Summary

1. **Slope of a Tangent Line:** $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
2. **Critical Points:** Solve $f'(x) = 0$ or where $f'(x)$ is undefined.
3. **Maxima/Minima with Second Derivative Test:**
 - $f''(x) > 0$ indicates a local minimum.
 - $f''(x) < 0$ indicates a local maximum.
4. **Mean Value Theorem (MVT):** $f'(c) = \frac{f(b) - f(a)}{b - a}$ for $c \in (a, b)$.
5. **L'Hôpital's Rule:** $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$ for indeterminate forms.
6. **Optimization:** Use $f'(x) = 0$ and test for maxima/minima within constraints.

These formulas and concepts form the foundation for solving a variety of practical problems, from determining the optimal dimensions of a shape to finding maximum profit or minimum cost in economics and physics.