Solution

KINETIC THEORY OF GASES

Class 11 - Physics

1. At low temperature and high pressure, the intermolecular attractions become appreciable. Moreover, the volume occupied by the gas molecules cannot be neglected in comparison to the volume of the gas. Hence the real gases show large deviations from ideal gas behaviour.
2. For O_2, $V_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3R\times300}{32}}$
   For SO_2, $V_{\text{rms}} = \sqrt{\frac{3RT}{M_0}} = \sqrt{\frac{3R\times T}{64}}$
   As V_0 = V
   $\therefore \sqrt{\frac{3RT}{64}} = \sqrt{\frac{3R\times300}{32}}$
   T = 600t = 600 - 273 = 327°C
3. An ideal gas is one that obeys the gas laws at all values of temperature and pressure. Its main characteristics are
   i. The size of the molecules is negligibly small.
   ii. There is no force of attraction or repulsion amongst its molecules.
4. From kinetic theory, $P = \frac{1}{3}\frac{M}{V}\overline{v^2}$
   But $\overline{v^2} \propto T \therefore P \propto \frac{MT}{V}$
   As both T and V remain unchanged but mass M is doubled, so the pressure of the mixture gets doubled i.e. it is equal to 2P.
5. The lower layers of earth's atmosphere reflect infrared radiations from earth back to the surface of earth. Thus the heat radiations received by the earth from the sun during the day are kept trapped by the atmosphere. If atmosphere of earth were not there, its surface would become too cold to live.
6. a. Average velocity of an ideal gas molecule is given by , $v_{av} = \bar{v} = \sqrt{\frac{8N_AK_BT}{\pi m}} = \sqrt{\frac{8RT}{\pi m}} = \sqrt{\frac{8PV}{\pi m}}$
   (As the temperature and pressure are same in this question.)
   Hence $v_{av} = \bar{v} \propto \frac{1}{\sqrt{m}}$
   Since $m_A > m_B > m_C, \therefore v_C > v_B > v_A$. Again velocity of the gas molecules will affect the KE more than mass of the molecules. So, average KE of molecules in decreasing order is given by $KE_C > KE_B > KE_A$
   b.
   Again we know that rms velocity of an ideal gas molecule is given by, $v_{rms} = \sqrt{\frac{3K_BT}{m}}$
   if Pressure, Temperature are constant then
   $\therefore v_{rms} \propto \frac{1}{\sqrt{m}}$
   Here also since $m_A > m_B > m_c$ (given)
   $\therefore (V_{rms})_C > (V_{rms})_B > (V_{rms})_A$
7. Length of the narrow bore is given by, L = 1 m = 100 cm
   Length of the mercury thread is, l = 76 cm
   Length of the air column between mercury and the closed-end, l_a = 15 cm
   Since the bore is held vertically in air with the open end at the bottom, the mercury length that occupies the air space is: 100 - (76 + 15) = 9 cm
   Hence, the total length of the air column is given by = 15 + 9 = 24 cm
   Let h cm of mercury flow out as a result of atmospheric pressure.
   ∴Length of the air column in the bore is = 24 + h cm
   And, length of the mercury column is given by = 76 - h cm
   Initial pressure is given by, P_1 = 76 cm of mercury
   Initial volume is given by, V_1 = 15 cm^3
   Final pressure is given by, P_2 = 76 – (76 – h) = h cm of mercury
   Final volume is given by, V_2 = (24 + h) cm^3
   Temperature remains constant throughout the process. Hence,
   $\therefore P_1V_1 = P_2V_2$
   76 $\times$ 15 = h (24 + h)

$h^2 + 24h - 1140 = 0$

$\therefore h = 23.8cm\ or - 47.8cm$

Height cannot be negative. Hence, 23.8 cm of mercury will flow out from the bore and 52.2 cm of mercury will remain in it. The length of the air column will be 24 + 23.8 = 47.8 cm.

8. According to Avogadro's law, under similar conditions of temperature and pressure equal volumes of all gases contain an equal number of molecules.
   Consider equal volumes (say V each) of two gases A and B being at the same temperature T and having same pressure P.
   According to kinetic theory, we have
   $P_1 = \frac{1}{3}\frac{m_1 N_1 \bar{v}_1^2}{V}$ and $P_2 = \frac{1}{3}\frac{m_2 N_2 \bar{v}_2^2}{V}$
   As $P_1 = P_2$,
   Hence $\frac{1}{3}\frac{m_1 N_1 \bar{v}_1^2}{V} = \frac{1}{3}\frac{m_2 N_2 \bar{v}_2^2}{V}$, where m_1 and m_2 are the molecular masses and N_1 and N_2 number of molecules present in two gases.
   $\Rightarrow m_1 N_1 \bar{v}_1^2 = m_2 N_2 \bar{v}_2^2$ ....(i)
   Again at given temperature T, the mean kinetic energy per molecule of a gas is constantly having the same value for all gases. Hence, we have
   $\frac{1}{2}m_1\bar{v}_1^2 = \frac{1}{2}m_2\bar{v}_2^2$ or $m_1\bar{v}_1^2 = m_2\bar{v}_2^2$ ...(ii)
   Dividing (i) by (ii), we get $N_1 = N_2$
   Thus, the number of molecules in two gases is equal in equal volumes provided that pressure and temperature are equal. It is the result of Avogadro's law.
9. For ideal gas equation
   $PV = \mu RT$
   For gasses in chamber 1 and 2
   $P_1V_1 = \mu_1 RT_1$
   $P_2V_2 = \mu_2 RT_2$
   P_1 = 1atm, P_2 = 2atm
   V_1 = 2L, V_2 = 3L
   T_1 = T, T_2 = T
   $\mu_1 = 4, \mu_2 = 5$
   When partition between gases removed then
   $\mu = \mu_1 + \mu_2$ and $V = V_1 + V_2$
   By the kinetic theory of gases
   The kinetic translational energy $PV = 2/3E$ per mole
   So the KE (translational) by gas of μ_1 moles
   $P_1V_1 = \frac{2}{3}\mu_1 E_1,$ and $\mu = \mu_1 + \mu_2$
   Adding both above
   $P_1V_1 + P_2V_2 = \frac{2}{3}\mu_1 E_1 + \frac{2}{3}\mu_2 E_2$
   Or $\mu_1 E_1 + \mu_2 E_2 = \frac{3}{2}(P_1V_1 + P_2V_2)$
   Combined effect, PV=$\frac{2}{3}E_{Total}$ per mole =$\frac{3}{2}\mu E$ per mole
   $P(V_1 + V_2) = \frac{2}{3}[\frac{3}{2}(P_1V_1 + P_2V_2)]$
   $P = \frac{P_1V_1 + P_2V_2}{V_1 + V_2} = \frac{1.00\times1.0 + 2.00\times3.0}{2.0+3.0}atm$
   $P = \frac{2+6}{5} = \frac{8}{5}$ = 1.6 atm
10. As, we know that the value of mean free path(λ) of the molecules of a gas is given by $\lambda = \frac{1}{\sqrt{2}\pi n d^2}$
    Here, n = number of gas molecules present per unit volume of given gas and d = diameter of the given molecule
    i. Effect of temperature : As temperature of a gas is increased at constant pressure, volume of the gas increases and the number of molecules per unit volume(n) decreases. In fact
       $n \propto \frac{1}{V}$ and $V \propto T$
       Thus, $n \propto \frac{1}{T}$
       The value of mean free path of the gas increases due to decrease in molecular numbers density i.e.$\lambda \propto \frac{1}{n} \propto T$. Thus the mean free path of a gas is directly proportional to its absolute temperature at constant pressure.

ii. Effect of pressure : At constant temperature, on increasing pressure, the volume V decreases, the molecular number density n increases and consequently, the mean free path decreases
i.e, $p \propto \frac{1}{V} \propto n$
$\therefore \lambda \propto \frac{1}{n}$
or $\lambda \propto \frac{1}{p}$
Thus the mean free path of a gas is inversely proportional to its pressure at constant temperature.

11. Initial volume of oxygen, V_1 = 30 litres = $30 \times 10^{-3} m^3$ (putting1 litre = 10^{-3} m^3)

Initial gauge pressure, P_1 = 15 atm = $15 \times 1.013 \times 10^5 Pa$ = 15.195 × 10^5 Pa(putting 1 atm = 1.013 × 10^5 Pa)

Initial temperature, T_1 = 27°C = (273 + 27)K = 300 K

Universal gas constant, R = 8.314 J mol^{-1} K^{-1}
Let the initial number of moles of oxygen gas in the cylinder be n_1.

The n_1 mole ideal gas equation is given as:

$P_1V_1 = n_1RT_1$
$\therefore n_1 = \frac{P_1V_1}{RT_1}$
$= \frac{15.195 \times 10^5 \times 30 \times 10^{-3}}{(8.314) \times 300} = \frac{151.95}{8.314}$ = 18.276
But, $n_1 = \frac{m_1}{M}$ (as, number of moles = total given mass of the gas ÷ molar mass of the gas)
Where,
m_1 = Initial mass of oxygen

M = Molecular mass of oxygen = 32 g
$\therefore$ m_1 = number of moles × molar mass of oxygen = n_1 M = 18.276 $\times$ 32 = 584.84 g

After some oxygen is withdrawn from the cylinder, the pressure and temperature reduces.
But as the cylinder of volume 30 litres is not changed, so the final volume of the gas will remain same.
$\therefore$Final volume, V_2 = 30 litres = $30 \times 10^{-3} m^3$

Final gauge pressure, P_2 = 11 atm = $11 \times 1.013 \times 10^3$Pa (given)

Final temperature, T_2 = 17°C = (273 + 17)K = 290 K(given)

Let n_2 be the number of moles of oxygen left in the cylinder.

Again the ideal gas equation for n_2 moles of Oxygen is given as:

$P_2V_2 = n_2RT_2$
$\therefore n_2 = \frac{P_2V_2}{RT_2}$
$= \frac{11.143 \times 10^5 \times 30 \times 10^{-3}}{8.314 \times 290} = \frac{11.143 \times 3}{8.314 \times 0.29}$ = 13.86
But, $n_2 = \frac{m_2}{M}$ (again using the formula, number of moles = total supplied mass of the gas ÷ molar mass of the gas)
Where,
m_2 is the mass of oxygen remaining in the cylinder

$\therefore$ m_2 = number of moles remaining × molar mass of Oxygen = n_2 × M = 13.86 $\times$ 32 = 453.1 g

The mass of oxygen taken out of the cylinder is given by the relation:
Change in mass, $\Delta m =$ Initial mass of oxygen in the cylinder - Final mass of oxygen in the cylinder
= m_1 - m_2

= 584.84 g - 453.1 g

= 131.74 g

= 0.131 kg

Therefore, 0.131 kg of oxygen is taken out of the cylinder.

12. a. **Moon** is considered **not** to **have an atmosphere** because it cannot absorb measurable quantities of radiation, **does not** appear layered or self-circulating, and requires constant replenishment due to the high rate at which its gases are lost to space.As acceleration due to gravity on moon is 1/6th of g on earth. So the escape velocity on moon
$V_{ez} = \sqrt{2gR} = 2.38 \text{km}/s$
M =Mass of hydrogen,As H_2 is lightest gas $m = 1.67 \times 10^{-24} kg$

$v_{rms=} = \sqrt{\frac{3K_BT}{m}} = \sqrt{\frac{3\times1.38\times10^{-23}\times300}{1.67\times10^{-24}}}$ = 2.72 km/s

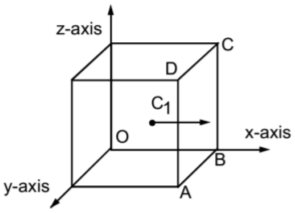
Due to small gravitational force and v_{rms} is greater than escape velocity so molecule of air can escape out.

As the distance of moon from sun is approximately equal to that of earth so the intensity of energy of sun reaches to moon is larger due to lower density of atmosphere, distance become smaller than earth when moon is towards sun during its rotation around earth.

Due to this (sun light), root-mean-square speed of molecule increase and some of them can speed up more than escape velocity and so probability of escaping out increased.

Hence over a long time moon has lost most of its atmosphere.

b. The temperature of atmosphere is due to the kinetic energy of air molecule. Due to lower atmospheric pressure at higher altitude molecules of air rises up so their potential energy increase, in turn, the kinetic energy decrease results the decrease in temperature. Due to lower atmospheric pressure at higher altitude, the gas expands and gives cooling effect and so decrease the temperature.

13. 

Consider an ideal gas contained in a cubical container shown in figure above, having each of side a and having volume V. Now, V = a^3 [$\because$(Side)^3 = volume of cube]

Let n = number of molecules of the gas contained in the container

m = Mass of each molecule

M = mn = Total mass of the gas contained

Consider one molecule is moving with velocity C_1 such that, $\vec{C}_1 = v_{1x}\hat{i} + v_{1y}\hat{j} + v_{1z}\hat{k}$

X-component of change in momentum of the molecule along +X axis = mv_{1x} - (- v_{1x}) = $2mv_{1x}$ in time t = $2a/v_{1x}$

Now rate of change of momentum (X component) = $(2mv_{1x})/t = \frac{mv_{1x}{}^2}{a}$, this is the X component of the force, F_{1x} by the molecule.

Now X component of pressure exerted by the molecule $P_{1x} = F_{1x}/a^2 = mv_{1x}{}^2/a^3$

Now taking all the molecules, net X component of pressure, $P_x = \frac{m}{a^3} \times \left(v_{1x}^2 + v_{2x}^2 + \ldots + v_{nx}^2\right)$ (for n number of molecules)

Similarly $P_y = \frac{m}{a^3}\left(v_{1y}^2 + v_{2y}^2 + \ldots. + v_{ny}^2\right)$ and

$P_z = \frac{m}{a^3}\left(v_{1z}^2 + v_{2z}^2 + \ldots. + v_{nz}^2\right)$

All three component of pressures are equal for all the gas molecules.

P = Total pressure of a single molecule = $\frac{P_x+P_y+P_z}{3}$

$= \frac{1}{3}\left[\frac{m}{a^3}\left(v_{1x}^2 + v_{2x}^2 + \ldots + v_{nx}^2\right) + \frac{m}{a^3}\left(v_{1y}^2 + v_{2y}^2 + \ldots + v_{nz}^2\right) + \frac{m}{a^3}\left(v_{1z}^2 + v_{2z}^2 + \ldots + v_{nz}^2\right)\right]$

$\therefore P = \frac{m}{3V}\left[C_1^2 + C_2^2 + \ldots. + C_n^2\right]$ where $C_1{}^2 = v_{1x}{}^2 + v_{1y}{}^2 + v_{1z}{}^2$, $C_2{}^2 = v_{2x}{}^2 + v_{2y}{}^2 + v_{2z}{}^2$,....., $C_n{}^2 = v_{nx}{}^2 + v_{ny}{}^2 + v_{nz}{}^2$

multiplying and dividing by n (total no of molecules of gas)

$\therefore P = \frac{m\times n}{3V}\left[\frac{C_1^2+C_2^2+\ldots.+C_n^2}{n}\right]$

$\Rightarrow P = \frac{M}{3V}C^2 = \frac{1}{3}\rho C^2$

$where\ C^2 = \frac{C_1^2+C_2^2+\ldots.+C_n^2}{n}$ or $C = \sqrt{\frac{C_1^2+C_2^2+\ldots.+C_n^2}{n}}$

C = r. m s. velocity of gas and ρ being the mass density of the gas.

14. **(b)** Assertion and reason both are correct statements but reason is not correct explanation for assertion.

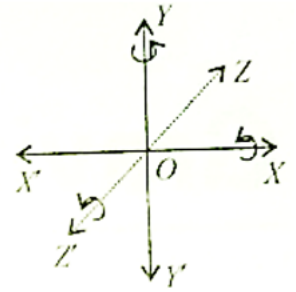
Explanation: Assertion and reason both are correct statements but reason is not correct explanation for assertion.

15. **(b)** Assertion and reason both are correct statements but reason is not correct explanation for assertion.

Explanation: According to Boyle's law, PV = constant, at a given temperature.

16. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Let the diatomic molecule has centre of mass at O. Such a molecules have three degree of freedom due to translational motion of the centre of mass. The molecule can rotate about the axes XX', YY', and ZZ' all are perpendicular to each other. The rotatory motion provides two more degrees of freedom. Thus the diatomic molecule has five degrees of freedom.



17. **(c)** Assertion is correct statement but reason is wrong statement.
Explanation: An undamped spring-mass system is the simplest vibration system with one degree of freedom.

18. **(a)** Assertion and reason both are correct statements and reason is correct explanation for assertion.
Explanation: Assertion and reason both are correct statements and reason is correct explanation for assertion.

19. **(c)** 1 + 2/n
Explanation: 1 + 2/n

20. **(c)** kT
Explanation: kT

21. **(c)** the average distance covered by a molecule between two successive collisions
Explanation: the average distance covered by a molecule between two successive collisions

22. **(b)** in random motion
Explanation: in random motion

23. **(b)** 4.148 joule
Explanation: 4.148 joule