Solution

TRIGONOMETRY

Class 11 - Mathematics

1.

(d) -$\frac{b}{a}$

Explanation:

Given: sin α + sin β = a ......(i)

cos α - cos β = b .....(ii)

Dividing (i) by (ii):

$\Rightarrow \frac{\sin \alpha+\sin B}{\cos \alpha-\cos B}$ = $\frac{a}{b}$

$\Rightarrow \frac{2 \sin\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right)}{-2 \sin\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right)}$ = $\frac{a}{b}$ [$\because$ sin A + sin B = 2 sin $\left(\frac{A+B}{2}\right)$ cos $\left(\frac{A-B}{2}\right)$ and cos A + cos B = -2 sin $\left(\frac{A+B}{2}\right)$ sin $\left(\frac{A-B}{2}\right)$]

$\Rightarrow \frac{\sin\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right)}{-\sin\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right)}$ = $\frac{a}{b}$

$\Rightarrow$ cot $\left(\frac{\alpha-\beta}{2}\right)$ = -$\frac{a}{b}$

$\Rightarrow \frac{1}{\cot\left(\frac{\alpha-\beta}{2}\right)}$ = $\frac{1}{-\frac{a}{b}}$

$\Rightarrow$ tan $\left(\frac{\alpha-\beta}{2}\right)$ = -$\frac{b}{a}$

2. **(a)** $\frac{-\sqrt{3}}{2}$

Explanation:

$\pi < x < \frac{3\pi}{2} \Rightarrow x$ lies in quadrant III and sin x < 0 in quadrant III.

sec x = - 2 $\Rightarrow$ cos $x = \frac{-1}{2}$

sin^{2}x = (1 - cos^{2}x) $= \left(1 - \frac{1}{4}\right) = \frac{3}{4}$

$\therefore \sin x = -\sqrt{\frac{3}{4}} = \frac{-\sqrt{3}}{2}$

3.

(d) cosec x

Explanation:

We have,

$\frac{2\left(\sin 2x+2 \cos^2 x-1\right)}{\cos x-\sin x-\cos 3x+\sin 3x}$

$= \frac{2(\sin 2x+\cos 2x)}{\cos x-\sin x-4 \cos^3 x+3 \cos x+3 \sin x-4 \sin^3 x}$

$= \frac{2(\sin 2x+\cos 2x)}{4 \cos x-4 \cos^3 x+2 \sin x-4 \sin^3 x}$

$= \frac{2(\sin 2x+\cos 2x)}{4 \cos x\left(1-\cos^2 x\right)+2 \sin x\left(1-2 \sin^2 x\right)}$

$= \frac{2(\sin 2x+\cos 2x)}{4 \cos x \sin^2 x+2 \sin x \cos 2x}$

$= \frac{2(\sin 2x+\cos 2x)}{2\times 2 \sin x \cos x \sin x+2 \sin x \cos 2x}$

$= \frac{2(\sin 2x+\cos 2x)}{2 \sin 2x \sin x+2 \sin x \cos 2x}$

$= \frac{2(\sin 2x+\cos 2x)}{2 \sin x(\sin 2x+\cos 2x)}$

$= \frac{1}{\sin x}$

= cosec x

4.

(d) sin 4β

Explanation:

It is given that $\tan\alpha = \frac{1}{7}$ and $\tan\beta = \frac{1}{3}$

Now, $\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta}$

= $\frac{2\times\frac{1}{3}}{1-\frac{1}{9}}$

= $\frac{\frac{2}{3}}{\frac{8}{9}}$

= $\frac{3}{4}$

$\therefore \tan(\alpha+2\beta) = \frac{\tan\alpha+\tan 2\beta}{1-\tan\alpha\tan 2\beta}$

= $\frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\times\frac{3}{4}}$ = $\frac{\frac{25}{28}}{\frac{25}{28}}$

= 1

$\tan(\alpha+2\beta)$ = 1 = tan $\frac{\pi}{4}$

$\Rightarrow \alpha+2\beta = \frac{\pi}{4}$

$\Rightarrow \alpha = \frac{\pi}{4} - 2\beta$

$\Rightarrow 2\alpha = \frac{\pi}{2} - 4\beta$

$\Rightarrow \cos 2\alpha$ = $\cos\left(\frac{\pi}{2} - 4\beta\right)$ = $\sin 4\beta$

$\therefore \cos 2\alpha = \sin 4\beta$

5. **(a)** $-\sqrt{3}$

Explanation:

$\tan\left(\frac{-16\pi}{3}\right) = -\tan\frac{16\pi}{3}$ $[\because \tan(-\theta) = -\tan\theta]$

$= -\tan\left(5\pi+\frac{\pi}{3}\right) = -\tan\frac{\pi}{3} = -\sqrt{3}$ $[\because \tan(n\pi+\theta) = \tan\theta]$

6.

(d) tan 3A tan 2A tan A

Explanation:

tan 3A = tan(2A + A)

tan 3A = $\frac{\tan 2\text{A}+\tan\text{A}}{1-\tan 2\text{A}\cdot\tan\text{A}}$

$\Rightarrow$ tan 3A(1 - tan 2A.tan A) = tan 2A + tan A

$\Rightarrow$ tan 3A - tan 3A.tan 2A.tan A = tan 2A + tan A

$\Rightarrow$ tan 3A - tan 2A - tan A = tan 3A.tan 2A.tan A

7.

(c) -1 and 1

Explanation:

The extremum values of sin θ are -1 and 1.

8.

(c) sin 10x sin 2x

Explanation:

$\sin^2 6x - \sin^2 4x$ = sin(6x + 4x)sin (6x- 4x) [$\because \sin^2 x - \sin^2 y$ = sin(x + y) sin(x - y)]

= sin 10x sin 2x.

9.

(d) GP

Explanation:

cot $\frac{\pi}{3} = \frac{1}{\sqrt{3}}$, cot $\frac{\pi}{4} = 1$, cot $\frac{\pi}{6} = \sqrt{3}$

Now, $\frac{1}{\sqrt{3}}, 1, \sqrt{3}$ are in GP with common ratio $\sqrt{3}$.

10.

(b) 2 cos θ

Explanation:

$\sqrt{2+\sqrt{2+2\cos 4\theta}} = \sqrt{2+\sqrt{2(1+\cos 4\theta)}} = \sqrt{2+\sqrt{4\cos^2 2\theta}} = \sqrt{2+2\cos 2\theta}$

$= \sqrt{2(1+\cos 2\theta)} = \sqrt{2\times 2\cos^2\theta} = \sqrt{4\cos^2\theta} = 2\cos\theta$

11. (a) - (ii), (b) - (iv), (c) - (i), (d) - (iii)
12. (a) - (ii), (b) - (iv), (c) - (i), (d) - (iii)
13. (a) - (ii), (b) - (iii), (c) - (i), (d) - (iv)
14. (a) - (iii), (b) - (i), (c) - (ii), (d) - (iv)
15. (a) - (iii), (b) - (iv), (c) - (ii), (d) - (i)
16. We know that the hour hand completes one rotation in 12 hours while the minute hand completes one rotation in 60 minutes.

$\therefore$ Angle traced by the hour hand in 12 hours = 360^o

$\Rightarrow$ Angle traced by the hour hand in 7 hours 20 minutes i.e., $\frac{22}{3}$ hours

= $\left(\frac{360}{12}\times\frac{22}{3}\right)^\circ$ = 220^o

Also, the angle traced by the minute hand in 60 minutes = 360^o

$\Rightarrow$ The angle traced by the minutes hand in 20 minutes = $\left(\frac{360}{60}\times 20\right)$ = 120^o

Hence, the required angle between two hands = 220^o - 120^o

= 100^o

17. L.H.S. = $\cos^2 x + \cos^2 y - 2\cos x \cos y \cos(x+y)$

$= \cos^2 x + \cos^2 y - \{\cos(x+y) + \cos(x-y)\}\cos(x+y)$

$= \cos^2 x + \cos^2 y - \cos^2(x+y) - \cos(x+y)\cos(x-y)$

$= \cos^2 x + \cos^2 y - \cos^2(x+y) - (\cos^2 x - \sin^2 y)$

$= \cos^2 x - \cos^2 y - \cos^2(x+y) - \cos^2 x + \sin^2 y$

$= \cos^2 y - \cos^2(x+y) + 1 - \cos^2 y$

$= 1 - \cos^2(x+y)$

$= \sin^2(x+y)$ = R.H.S.

Hence Proved

18. It is given that x lies in 3rd quadrant.

$\therefore \pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

$\Rightarrow \frac{x}{2}$ lies in 2nd quadrant

$\Rightarrow$ cos x < 0, sin $\frac{x}{2}$ > 0 and tan $\frac{x}{2}$ < 0

$\therefore \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

$\Rightarrow \cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}}$

$\Rightarrow \cos\frac{x}{2} = -\sqrt{\frac{1-\frac{1}{3}}{2}} = \frac{-1}{\sqrt{3}}$

and $\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

$\Rightarrow \sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}}$

$\Rightarrow \sin\frac{x}{2} = \sqrt{\frac{1+\frac{1}{3}}{2}} = \sqrt{\frac{2}{3}}$

$\therefore \tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\sqrt{\frac{2}{3}}}{\frac{-1}{\sqrt{3}}} = -\sqrt{2}$

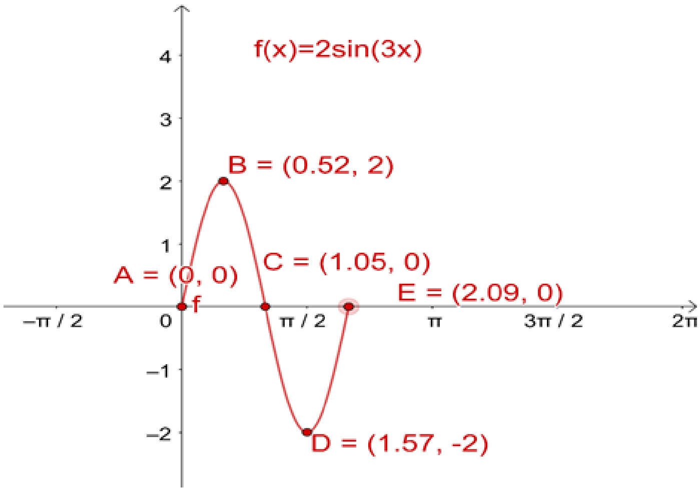
19. We know that g (x) = sin x is a periodic function with period π.

$\therefore$ h(x) = 2 sin 3x is a periodic function with period $\frac{2\pi}{3}$. So, we will draw the graph of h (x) = 2 sin 3x in the interval [0, $\frac{2\pi}{3}$]. The values of h(x) = 2 sin 3x at various points in [0, $\frac{2\pi}{3}$] are listed in the following table:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$

h(x) = 2 sin 3x	0	2	0	-2	0

By plotting the above points, we obtain the required curve.



20. According to the question, we can write,

LHS = 16 $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15}$

= $4\left(2\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\right)\left(2\cos\frac{4\pi}{15}\cos\frac{14\pi}{15}\right)$

= $4\left(\cos\frac{2\pi}{3}+\cos\frac{2\pi}{5}\right)\left(\cos\frac{6\pi}{5}+\cos\frac{2\pi}{3}\right)$

= $4\left(-\sin\frac{\pi}{6}+\sin\frac{\pi}{10}\right)\left(-\cos\frac{\pi}{5}-\sin\frac{\pi}{6}\right)$

= $4\left(-\frac{1}{2}+\frac{\sqrt{5}-1}{4}\right)\left(-\frac{\sqrt{5}+1}{4}-\frac{1}{2}\right)$

= $4\left(\frac{\sqrt{5}-3}{4}\right)\left(\frac{-\sqrt{5}-3}{4}\right)$

= $4\left(\frac{3-\sqrt{5}}{4}\right)\left(\frac{3+\sqrt{5}}{4}\right)$

= $\left(\frac{9-5}{4}\right)$

= 1

LHS = RHS

21. To prove: $\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{5\pi}{8}+\sin^2\frac{7\pi}{8}=2$

LHS = $\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{5\pi}{8}+\sin^2\frac{7\pi}{8}$

Identities used:

cos 2x = 1 – 2 $\sin^2$ x

$\Rightarrow$ 2 $\sin^2$ x = 1 - cos 2x

$\Rightarrow$ $\sin^2 x = \frac{1-\cos 2x}{2}$

Therefore,

$=\frac{1-\cos\frac{2\pi}{8}}{2}+\frac{1-\cos\frac{6\pi}{8}}{2}+\frac{1-\cos\frac{10\pi}{8}}{2}+\frac{1-\cos\frac{14\pi}{8}}{2}$

$=\frac{1-\cos\frac{2\pi}{8}}{2}+\frac{1-\cos\left(\pi-\frac{2\pi}{8}\right)}{2}+\frac{1-\cos\left(\pi+\frac{2\pi}{8}\right)}{2}+\frac{1-\cos\left(2\pi-\frac{2\pi}{8}\right)}{2}$ ... $\left\{\because \pi-\frac{2\pi}{8}=\frac{6\pi}{8};\pi+\frac{2\pi}{8}=\frac{10\pi}{8};2\pi-\frac{2\pi}{8}=\frac{14\pi}{8}\right\}$

$=\frac{1-\cos\frac{2\pi}{8}}{2}+\frac{1-\left(-\cos\frac{2\pi}{8}\right)}{2}+\frac{1-\left(-\cos\frac{2\pi}{8}\right)}{2}+\frac{1-\cos\frac{2\pi}{8}}{2}$ ... {∵ cos (π - θ) = - cos θ, cos (π + θ) = - cos θ and cos(2π - θ) = cos θ}

$=\frac{1-\cos\frac{2\pi}{8}}{2}+\frac{1+\cos\frac{2\pi}{8}}{2}+\frac{1+\cos\frac{2\pi}{8}}{2}+\frac{1-\cos\frac{2\pi}{8}}{2}$

$=2\times\frac{1-\cos\frac{2\pi}{8}}{2}+2\times\frac{1+\cos\frac{2\pi}{8}}{2}$

= 1 - $\cos\frac{2\pi}{8}$ + 1 + $\cos\frac{2\pi}{8}$

= 2

LHS = RHS

Hence proved.