



# TRIGONOMETRY

## SECTION A



### 1 MARK QUESTIONS

#### Measurement of Angles

1. (i) Find the radian measures corresponding to the following degree measures  
(a)  $25^\circ$  (b)  $-47^\circ 30'$   
(ii) Find the degree measures corresponding to the following radian measures

(a)  $\frac{11}{16}$  (b)  $-4$

2. Find the values of

(i)  $\sin 765^\circ$  (ii)  $\tan \frac{19\pi}{3}$

3. Find the values of

(i)  $\sin 75^\circ$  (ii)  $\cos 15^\circ$

4. Find the principal solutions of the following equations.

(i)  $\sin x = 1$

(ii)  $\sin x = \frac{1}{2}$

(iii)  $\sin x = \frac{\sqrt{3}}{2}$

(iv)  $\cos x = \frac{\sqrt{3}}{2}$

(v)  $\sec x = 2$

(vi)  $\cos x = -\frac{1}{2}$

(vii)  $\tan x = \frac{1}{\sqrt{3}}$

(viii)  $\tan x = 1$

(ix)  $\tan x = \sqrt{3}$

(x)  $\operatorname{cosec} x = -\frac{2}{\sqrt{3}}$



### 4 MARKS QUESTIONS

5. (i) A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second.  
(ii) In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.  
(iii) Find the angle subtended at the centre of a circle of radius 4 feet by an arc of length 1 foot.  
(iv) Find the angle in degrees subtended at the centre of a circle by an arc whose length is 2.2 times the radius.  
(v) Express in radians the angle subtended at the centre of a circle by an arc whose length is 15 feet, the radius of the circle being 25 feet.  
(vi) What is the ratio of the radii of two circles, at the centre of which two arcs of the same length subtend angles  $60^\circ$  and  $75^\circ$ ?  
(vii) Find the radius of a circle in which a central angle of  $45^\circ$  intercepts an arc of 176 cm.

- (viii) The large hand of a big clock is 70 cm long. How many centimetres does its extremity move in 6 minutes time?
- (ix) A horse is tied to a post by a rope. If the horse moves along a circular path always keeping the rope tight and describes 88 metres when it has traced out  $72^\circ$  at the centre, find the length of the rope?
- (x) Find the angle between the minute hand and the hour hand of a clock when the time is 7.20.
- (xi) At what distance does a man, whose height is 6 feet subtend an angle of  $10'$ ?
- (xii) If the circumference of a circle be divided into five parts which are in A.P., and if the greatest part be six times the least, find in radians the magnitudes of the angles that the parts subtend at the centre of the circle.
- (xiii) Find the length which at a distance of one mile will subtend an angle of  $1'$  at the eye.
6. Find the values of other five trigonometric functions

(i)  $\cos x = -\frac{1}{2}$ ,  $x$  lies in 3rd quadrant

(ii)  $\sin x = \frac{3}{5}$ ,  $x$  lies in 2nd quadrant

7. Prove that

(i)  $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = -\frac{1}{2}$

(ii)  $2 \sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} = \frac{3}{2}$

### Identities

8. Prove the following statements.

(i)  $\cos^4 A - \sin^4 A + 1 = 2 \cos^2 A$

(ii)  $\sqrt{\sec^2 A + \operatorname{cosec}^2 A} = \tan A + \cot A$

(iii)  $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$

(iv)  $\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \operatorname{cosec} A - \cot A$

(v)  $\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A} = 2 \operatorname{cosec} A$

(vi)  $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2 \sec^2 A$

(vii)  $(\sec A + \cos A)(\sec A - \cos A) = \tan^2 A + \sin^2 A$

(viii)  $\frac{1}{\sec A + \tan A} = \sec A - \tan A$

(ix)  $\frac{\sec A - \tan A}{\sec A + \tan A} = 1 - 2 \sec A \tan A + 2 \tan^2 A$

(x)  $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} = \sec A \operatorname{cosec} A +$

$\frac{1}{2(\sin^6 A + \cos^6 A) - 3(\sin^4 A + \cos^4 A) + 1} = 0$

(xii)  $\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2$

(xiii)  $\sec^2 A \operatorname{cosec}^2 A = \tan^2 A + \cot^2 A + 2$

(xiv)  $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

(xv)  $\frac{1}{\operatorname{cosec} A - \cot A} + \frac{1}{\operatorname{cosec} A + \cot A} = \frac{2}{\sin A}$

9. Prove the following statements.

(i)  $\sin(A + B) \sin(A - B) = \sin^2 A - \sin^2 B$

(ii)  $\cos(A + B) \cos(A - B) = \cos^2 A - \sin^2 B$

(iii)  $\sin(45^\circ + A) \cos(45^\circ - B) - \sin(45^\circ - B) \cos(45^\circ + A) = \sin(A + B)$

(iv)  $\sin(45^\circ + A) \cos(45^\circ - B) + \cos(45^\circ + A) \sin(45^\circ - B) = \cos(A - B)$

(v)  $\sin 105^\circ + \cos 105^\circ = \cos 45^\circ$

(vi)  $\sin \{(n + 1)A\} \sin \{(n - 1)A\} + \cos \{(n + 1)A\} \cos \{(n - 1)A\} = \cos 2A$

$$(vii) \sin \{(n+1)A\} \sin \{(n+2)A\} + \cos \{(n+1)A\} \cos \{(n+2)A\} = \cos A$$

$$(viii) \sin 2A = 2 \sin A \cos A$$

$$(ix) \cos 2A = \cos^2 A - \sin^2 A \\ = 2 \cos^2 A - 1 \\ = 1 - 2 \sin^2 A$$

$$(x) \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$(xi) \tan 9^\circ = \frac{\cos 36^\circ - \sin 36^\circ}{\cos 36^\circ + \sin 36^\circ}$$

$$(xii) \sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$(xiii) \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$(xiv) \sin 3A = 3 \sin A - 4 \sin^3 A$$

$$(xv) \cos 3A = 4 \cos^3 A - 3 \cos A$$

10. Prove the following statements.

$$(i) \frac{\sin 7A - \sin 5A}{\cos 7A + \cos 5A} = \tan A$$

$$(ii) \frac{\sin A + \sin 3A}{\cos A + \cos 3A} = \tan 2A$$

$$(iii) \frac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ} = \frac{1}{\sqrt{3}}$$

$$(iv) \frac{\sin 7A - \sin A}{\sin 8A - \sin 2A} = \cos 4A \sec 5A$$

$$(v) \frac{\cos 2B - \cos 2A}{\sin 2B + \sin 2A} = \tan (A - B)$$

$$(vi) \frac{\sin A + 2 \sin 3A + \sin 5A}{\sin 3A + 2 \sin 5A + \sin 7A} = \frac{\sin 3A}{\sin 5A}$$

$$(vii) \cos 3A + \cos 5A + \cos 7A + \cos 15A = 4 \cos 4A \cos 5A \cos 6A$$

$$(viii) \sin A + \sin 2A + \sin 4A + \sin 5A = 4 \cos \frac{A}{2}$$

$$\cos \frac{3A}{2} \sin 3A$$

$$(ix) \sin 10^\circ + \sin 50^\circ - \sin 70^\circ = 0$$

$$(x) \sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ \\ - \sin 70^\circ - \sin 80^\circ = 0$$

$$(xi) \sin \frac{A}{2} \sin \frac{7A}{2} + \sin \frac{3A}{2} \sin \frac{11A}{2} = \sin 2A \sin 5A$$

$$(xii) \cos 2A \cos \frac{A}{2} - \cos 3A \cos \frac{9A}{2} \\ = \sin 5A \sin \frac{5A}{2}$$

$$(xiii) \sin A \sin (A + 2B) - \sin B \sin (B + 2A) = \sin^2 A - \sin^2 B$$

$$(xiv) (\sin 3A + \sin A) \sin A + (\cos 3A - \cos A) \cos A = 0$$

$$(xv) \cos (36^\circ - A) \cos (36^\circ + A) + \cos (54^\circ + A) \cos (54^\circ - A) = \cos 2A$$

11. Prove the following statements.

$$(i) \tan A + \cot A = 2 \operatorname{cosec} 2A$$

$$(ii) \cot A - \tan A = 2 \cot 2A$$

$$(iii) \operatorname{cosec} 2A + \cot 2A = \cot A$$

$$(iv) \frac{1 - \cos A + \cos B - \cos(A + B)}{1 + \cos A - \cos B - \cos(A + B)} \\ = \tan \frac{A}{2} \cot \frac{B}{2}$$

$$(v) \frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$$

$$(vi) \frac{1 + \tan^2 (45^\circ - A)}{1 - \tan^2 (45^\circ - A)} = \operatorname{cosec} 2A$$

$$(vii) \tan \left( \frac{\pi}{4} + A \right) - \tan \left( \frac{\pi}{4} - A \right) = 2 \tan 2A$$

$$(viii) \cot (A + 15^\circ) - \tan (A - 15^\circ) = \frac{4 \cos 2A}{1 + 2 \sin 2A}$$

$$(ix) \frac{1 + \sin A - \cos A}{1 + \sin A + \cos A} = \tan \frac{A}{2}$$

$$(x) \frac{\sin \{(n+1)A\} - \sin \{(n-1)A\}}{\cos \{(n+1)A\} + 2 \cos nA \\ + \cos \{(n-1)A\}} =$$

$$= \tan \frac{A}{2}$$

$$(xi) \frac{\sin \{(n+1)A\} + 2 \sin nA}{\cos \{(n-1)A\} - \cos \{(n+1)A\}}$$

$$= \cot \frac{A}{2}$$

$$(xii) \sec^2 A (1 + \sec 2A) = 2 \sec 2A$$

$$(xiii) \sqrt{2 + \sqrt{2 + 2 \cos 4A}} = 2 \cos A$$

$$(xiv) \tan (A - B) + \tan (B - C) + \tan (C - A) = \tan (A - B) \tan (B - C) \tan (C - A)$$

$$(xv) \sin A + \sin B + \sin C - \sin (A + B + C) = 4 \sin \frac{A+B}{2} \sin \frac{B+C}{2} \sin \frac{C+A}{2}$$

12. Prove the following statements.

$$(i) \tan 15^\circ = 2 - \sqrt{3}$$

$$(ii) \tan 75^\circ = 2 + \sqrt{3}$$

$$(iii) \sin 22 \frac{1^\circ}{2} = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$

$$(iv) \cos 22 \frac{1^\circ}{2} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$

$$(v) \sin 18^\circ = \frac{\sqrt{5}-1}{4}$$

$$(vi) \cos 36^\circ = \frac{\sqrt{5}+1}{4}$$

$$(vii) \sin 7 \frac{1^\circ}{2} = \frac{\sqrt{4-\sqrt{6}-\sqrt{2}}}{2\sqrt{2}}$$

$$(viii) \cos 7 \frac{1^\circ}{2} = \frac{\sqrt{4+\sqrt{6}+\sqrt{2}}}{2\sqrt{2}}$$

$$(ix) \tan 11 \frac{1^\circ}{4} = \sqrt{4+2\sqrt{2}} - (\sqrt{2}+1)$$

$$(x) \sin^2 72^\circ - \sin^2 60^\circ = \frac{\sqrt{5}-1}{8}$$

$$(xi) \sin \frac{\pi}{5} \sin \frac{2\pi}{5} \sin \frac{3\pi}{5} \sin \frac{4\pi}{5} = \frac{5}{16}$$

$$(xii) \sin \frac{\pi}{10} + \sin \frac{13\pi}{10} = -\frac{1}{2}$$

$$(xiii) \sin \frac{\pi}{10} \cdot \sin \frac{13\pi}{10} = -\frac{1}{4}$$

$$(xiv) \sin^2 24^\circ - \sin^2 6^\circ = \frac{\sqrt{5}-1}{8}$$

$$(xv) \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{14\pi}{15} = \frac{1}{16}$$

13. Find the general solutions of the following equations.

$$(i) \sin \alpha = \sin \beta$$

$$(ii) \cos \alpha = \cos \beta$$

$$(iii) \tan \alpha = \tan \beta$$

$$(iv) \sin x = \frac{1}{\sqrt{2}}$$

$$(v) \cos x = \frac{1}{\sqrt{2}}$$

$$(vi) \tan x = 2 - \sqrt{3}$$

$$(vii) \cos 4x = \cos 2x$$

$$(viii) \cos 3x = \sin 2x$$

$$(ix) \sin mx + \sin nx = 0$$

$$(x) \sin x - \cos x = \sqrt{2}$$

$$(xi) \sqrt{3} \sin x - \cos x = \sqrt{2}$$

$$(xii) \sqrt{3} \cos x + \sin x = \sqrt{2}$$

$$(xiii) \sin x + \sin 2x + \sin 3x = 0$$

$$(xiv) \cos x + \cos 2x + \cos 3x = 0$$

$$(xv) \tan x + \tan 2x + \tan 3x = 0$$

$$(xvi) \tan x + \tan 2x + \tan x \tan 2x = 1$$

$$(xvii) \tan x + \tan 2x + \tan 3x = \tan x \tan 2x \tan 3x$$

$$(xviii) \tan^2 x + \cot^2 x = 2$$

$$(xix) \cot^2 x + 3 \operatorname{cosec} x + 3 = 0$$

$$(xx) 2 \tan x - \cot x + 1 = 0$$



## 6 MARKS QUESTIONS

## Conditional Identities

14. In a  $\triangle ABC$ , prove that

- (i)  $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$
- (ii)  $\sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \sin C$
- (iii)  $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$
- (iv)  $\sin A + \sin B - \sin C = 4 \sin \frac{A}{2} \sin \frac{B}{2} \cos \frac{C}{2}$
- (v)  $\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$
- (vi)  $\cos A + \cos B - \cos C = -1 + 4 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$
- (vii)  $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2 \cos A \cos B \cos C$
- (viii)  $\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2 \sin A \sin B \cos C$
- (ix)  $\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} + \sin^2 \frac{C}{2} = 1 - 2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$
- (x)  $\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} - \sin^2 \frac{C}{2} = 1 - 2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$
- (xi)  $\cos^2 \frac{A}{2} + \cos^2 \frac{B}{2} + \cos^2 \frac{C}{2} = 2 + 2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$
- (xii)  $\cos^2 \frac{A}{2} + \cos^2 \frac{B}{2} - \cos^2 \frac{C}{2} = 2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$
- (xiii)  $\tan A + \tan B + \tan C = \tan A \tan B \tan C$
- (xiv)  $\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$
- (xv)  $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$



## HOTS-based Questions



- 15.(i) If  $\cos A - \sin A = \sqrt{2} \sin A$ , prove that  $\cos A + \sin A = \sqrt{2} \cos A$
- (ii) If  $\sin A = \frac{m^2 + 2mn}{m^2 + 2mn + 2n^2}$ , prove that  $\tan A = \frac{m^2 + 2mn}{2mn + 2n^2}$
- (iii) If  $x$  is real, prove that the equation  $\sin A = x + \frac{1}{x}$  is impossible.
- (iv) Show that the equation  $\sec^2 A = \frac{4xy}{(x+y)^2}$  is only possible when  $x = y$ .
- (v) If  $m = \tan A + \sin A$ ,  $n = \tan A - \sin A$ , show that  $m^2 - n^2 = 4 \sqrt{nm}$
- (vi) If  $2 \tan A = 3 \tan B$ , prove that  $\tan(A - B) = \frac{\sin 2B}{5 - \cos 2B}$

(vii) If  $\sin A + \sin B = a$ ,  $\cos A + \cos B = b$

$$\text{Prove that } \tan \frac{A-B}{2}$$

$$= \sqrt{\frac{4 - a^2 - b^2}{a^2 + b^2}}$$

(viii) If  $\cot A + \operatorname{cosec} A = 5$ , prove that

$$\cos A = \frac{12}{13}$$

(ix) If  $\sec^2 A = 2 + 2 \tan A$ , prove that  $\tan A = \pm\sqrt{2} + 1$

(x) If  $\cos A + \cos B = \frac{1}{3}$  and  $\sin A + \sin B = \frac{1}{4}$ , prove that  $\tan \frac{A+B}{2} = \frac{3}{4}$

## SECTION B



### 1 MARK QUESTIONS

1. (i) Find the radian measures corresponding to following degree measures.

(a)  $300^\circ$       (b)  $450^\circ$

(ii) Find the degree measures corresponding to the following radian measures.

(a)  $\frac{2\pi}{3}$       (b)  $\frac{12}{13}$

2. Find the values of

(i)  $\cos 765^\circ$       (ii)  $\sin \frac{31\pi}{3}$

3. Find the principal solution of

(i)  $\sin x = \frac{1}{\sqrt{2}}$       (ii)  $\cos x = \frac{1}{2}$

(iii)  $\tan x = \frac{1}{\sqrt{3}}$       (iv)  $\sec x = \sqrt{2}$



### 4 MARKS QUESTIONS

4. (i) Find the angle subtended at the centre of a circle of diameter 50 cm by an arc of 11 cm.

(ii) A wire in the form of a semi-circle of radius 8 cm, is bent and placed along the cir-

cumference of a circular hoop. It subtends an angle of  $30^\circ$  at the centre of the hoop; find the radius of the hoop.

(iii) Modinagar is 64 kilometres from Delhi. Find, to the nearest second, the angle subtended at the centre of the earth by the arc joining these two towns, earth being regarded as a sphere of 6400 kilometres of radius.

(iv) When the moon is setting at any given place, the angle that is subtended at its centre by the radius of the earth passing through the given place is 57 minutes. If the earth's radius be 3960 miles, find approximately the distance of the moon.

(v) A wheel makes 270 revolutions in one minute, through how many radians does it turn in one second?

(vi) Assuming the average distance of the earth from the sun to be 92,500,000 miles and the angle subtended by the sun at the eye of a person on the earth to be  $32'$ , find the sun's diameter.

(vii) The moon's distance from the earth is 360000 km and its diameter subtends an angle of  $31'$  at the eye of the observer on the earth. Find the radius of the moon.

(viii) Assuming that a person of a normal sight can read print at such distance that the letters subtend an angle of  $5'$  at his eye. Find what is the height of the letters that he can read at a distance of 12 feet.