

Trig Class 11 NCERT



Lecture Notes: Trigonometric Functions

1. Introduction to Trigonometric Functions

Trigonometric functions relate the angles of a triangle to the lengths of its sides. They are fundamental in the study of periodic phenomena, waves, and oscillations.

Basic Trigonometric Functions:

- **Sine (sin):** $\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$
- **Cosine (cos):** $\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$
- **Tangent (tan):** $\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$

Example Problem:

Find the sine, cosine, and tangent of a 30° angle in a right triangle.

Solution:

For a 30° angle:

- $\sin 30^\circ = \frac{1}{2}$
- $\cos 30^\circ = \frac{\sqrt{3}}{2}$
- $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

2. Signs of Trigonometric Functions

The signs of trigonometric functions depend on the quadrant in which the angle is located.

Quadrants:

- **1st Quadrant (0° to 90°):** All functions are positive.
- **2nd Quadrant (90° to 180°):** Sine is positive, others are negative.
- **3rd Quadrant (180° to 270°):** Tangent is positive, others are negative.
- **4th Quadrant (270° to 360°):** Cosine is positive, others are negative.

Example Problem:

Determine the signs of $\sin 120^\circ$, $\cos 240^\circ$, and $\tan 330^\circ$.

Solution:

- $\sin 120^\circ$: 2nd Quadrant, positive.
- $\cos 240^\circ$: 3rd Quadrant, negative.
- $\tan 330^\circ$: 4th Quadrant, negative.

3. Domain and Range of Trigonometric Functions

Sine and Cosine Functions:

- **Domain:** All real numbers $(-\infty, \infty)$
- **Range:** $[-1, 1]$

Tangent Function:

- **Domain:** All real numbers except $(2n + 1)\frac{\pi}{2}$, where n is an integer.
- **Range:** All real numbers $(-\infty, \infty)$

Example Problem:

Find the domain and range of $\sin x$.

Solution:

- Domain: $(-\infty, \infty)$
- Range: $[-1, 1]$

4. Trigonometric Functions of Sum and Difference of Two Angles

These formulas help simplify the expressions involving trigonometric functions of sums and differences.

Sum and Difference Formulas:

- $\sin(a + b) = \sin a \cos b + \cos a \sin b$
- $\sin(a - b) = \sin a \cos b - \cos a \sin b$
- $\cos(a + b) = \cos a \cos b - \sin a \sin b$
- $\cos(a - b) = \cos a \cos b + \sin a \sin b$
- $\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$
- $\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$

Example Problem:

Calculate $\sin 75^\circ$ using the sum formula.

Solution:

$$\sin 75^\circ = \sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

$$= \left(\frac{\sqrt{2}}{2}\right) \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right) \left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

