

# 3D Geometry formulae

## CBSE Class 12 – 3D Geometry Important Formulae

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### 1. Direction Cosines (DCs) and Direction Ratios (DRs)

- If a line makes angles  $\alpha$ ,  $\beta$ ,  $\gamma$  with x-, y-, z-axis respectively:

$$l = \cos \alpha, \quad m = \cos \beta, \quad n = \cos \gamma$$

- Relation between DCs:

$$l^2 + m^2 + n^2 = 1$$

- If DRs are  $a$ ,  $b$ ,  $c$ , then

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, \quad m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \quad n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

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### 2. Angle between two lines

If DRs of lines are  $(a_1, b_1, c_1)$  and  $(a_2, b_2, c_2)$ :

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

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### 3. Equation of a line

- Passing through  $(x_1, y_1, z_1)$  with DRs  $a$ ,  $b$ ,  $c$ :

**Symmetric form:**

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

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### 4. Shortest distance between two skew lines

If lines are:

$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$$

$$\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$$

Then shortest distance (SD):

$$SD = \frac{|(\vec{A}_2 - \vec{A}_1) \cdot (\vec{d}_1 \times \vec{d}_2)|}{|\vec{d}_1 \times \vec{d}_2|}$$

where  $\vec{A}_1 = (x_1, y_1, z_1)$ ,  $\vec{d}_1 = (l_1, m_1, n_1)$  etc.

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## 5. Equation of a plane

- Passing through  $(x_1, y_1, z_1)$  with normal vector  $\vec{n} = (a, b, c)$ :

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

- **General form:**

$$ax + by + cz + d = 0$$


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## 6. Angle between two planes

For planes  $a_1x + b_1y + c_1z + d_1 = 0$  and  $a_2x + b_2y + c_2z + d_2 = 0$ :

$$\cos \theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$


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## 7. Distance from a point to a plane

If plane is  $ax + by + cz + d = 0$  and point  $(x_1, y_1, z_1)$ :

$$\text{Distance} = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$


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## 8. Angle between a line and a plane

If line has DRs  $(l, m, n)$  and plane has normal  $(a, b, c)$ :

$$\sin \theta = \frac{|al + bm + cn|}{\sqrt{a^2 + b^2 + c^2} \sqrt{l^2 + m^2 + n^2}}$$

( $\theta$  = angle between line and plane)

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## 9. Condition of perpendicularity and parallelism

- **Two lines perpendicular:**  $a_1a_2 + b_1b_2 + c_1c_2 = 0$
- **Two lines parallel:**  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- **Two planes perpendicular:**  $a_1a_2 + b_1b_2 + c_1c_2 = 0$
- **Two planes parallel:**  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

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