Solution

SETS

Class 11 - Mathematics

1. Let B = {2, 4,8,16,32}

   All objects of the set are natural numbers that are powers of 2.

   $\therefore B = \{x : x = 2^n, n \in N \, and \; 1 \leqslant n \leqslant 5\}$
2. Finite set, Because, B = {2}
3. Disjoint sets have their intersections as ϕ

   A = {3, 4, 5, 6} and B = {2, 5, 7, 9} then A∩B={5}. So ,are pairs of disjoint sets.
4. Suppose A = {1, 2}

   B = {2, 3}

   C = {1, 3, 4}

   A ∩ B = {2}

   A ∩ C = {1}

   B ∩ C = {3}

   A ∩ B ∩ C = {2} ∩ {1, 3, 4} = ϕ

   Therefore the three sets are valid and satisfy the given conditions.
5. {b, c} is not a subset of given set. But it belongs to the given set.

   ∴ The correct form would be

   {b, c} ∈ {a, {b, c}}.
6. Given, (a -1, b + 5) = (2, 3)

   By the equality of sets, we get

   a - 1 = 2

   ⇒ a = 3

   and b + 5 = 3

   b = 3 - 5 = 2

   ∴ a = 3 and b = -2
7. The answer is A $\not\subset$ B

   Explanation: A $\not\subset$ B since 0 ∈ A and 0 ∉ B
8. The answer is C = (2, 6]

   C = {x : x ∈ R, 2 < x ≤ 6} is an open interval from 2 to 6, including 6 but excluding 2.
9. The statement is false.

   Take A = {1, 2} B = {2, 3}, C = {1, 2, 5} Now $A \not\subset B$ and $B \not\subset C$ but $A \subset C$
10. If A = {2, 4, 6, 8, 10, 12} and B = {3, 4, 5, 6, 7, 8, 10}

    Therefore,(B - A) = {3,5, 7}
11. Given: A = {$\frac{1}{x}$: x ∈ N and x < 8} and B = {$\frac{1}{2x}$: x ∈ N and x ≤ 4}

    According to the given conditions;

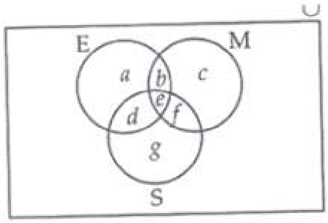
    $A = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}\right\}$ and $B = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}\right\}$

    Therefore,A ∩ B = $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}\right\}$
12. The interval [-23, 5) can be written in set builder form as $(x : x \in R, -23 \leqslant x < 5)$

13. Let the set of students who passed in Mathematics be M, the set of students who passed in English be E and the set of students who passed in Science be S.

Then $n(U) = 100$, $n(M) = 12$, $n(E) = 15$, $n(S) = 8$, $n(E \cap M) = 6$, $n(M \cap S) = 7$, $n(E \cap S) = 4$ and $n(E \cap M \cap S) = 4$

Let us draw a Venn diagram



According to the Venn diagram,

$n(E \cap S) = 4 \Rightarrow e = 4$

$n(E \cap M) = 6 \Rightarrow b + e = 6 \Rightarrow b + 4 = 6 \Rightarrow b = 2$

$n(M \cap S) = 7 \Rightarrow e + f = 7 \Rightarrow 4 + f = 7 \Rightarrow f = 3$

$n(E \cap S) = 4 \Rightarrow d + e = 4 \Rightarrow d + 4 = 4 \Rightarrow d = 0$

$n(E) = 15 \Rightarrow a + b + d + e = 15 \Rightarrow a + 2 + 0 + 4 = 15 \Rightarrow a = 9$

$n(M) = 12 \Rightarrow b + c + e + f = 12 \Rightarrow 2 + c + 4 + 3 = 12 \Rightarrow c = 3$

$n(S) = 8 \Rightarrow d + e + f + g = 8 \Rightarrow 0 + 4 + 3 + g = 8 \Rightarrow g = 1$

Hence we get,

i. Number of students who passed in English and Mathematics but not in Science, b = 2.
ii. Number of students who passed in Mathematics and Science but not in English, f = 3.
iii. Number of students who passed in Mathematics only, c = 3.
iv. Number of students who passed in more than one subject = b + e + d + f = 2 + 4 + 0 + 3 = 9.

14. L.H.S.

$(B \cup C) = \{x : x \in B \text{ or } x \in C\}$

$= \{2, 3, 4, 5, 6, 7\}$

$(A \cap (B \cup C)) = \{x : x \in A \text{ and } x \in (B \cup C)\}$

$= \{2, 4, 5\}$

R.H.S:

$(A \cap B) = \{x : x \in A \text{ and } x \in B\}$

$= \{2, 5\}$

$(A \cap C) = \{x : x \in A \text{ and } x \in C\}$

$= \{4, 5\}$

$(A \cap B) \cup (A \cap C) = \{x : x \in (A \cap B) \text{ and } x \in (A \cap C)\}$

$= \{2, 4, 5\}$. Hence proved.

15. Let $x \in A \cup (B - A)$

$\Rightarrow x \in A$ or $x \in (B - A)$

$\Rightarrow x \in A$ or $x \in B$ and $x \notin A$

$\Rightarrow x \in B$

$\Rightarrow x \in (A \cup B) [\because B \subset (A \cup B)]$

This is true for all $x \in A \cup (B - A)$
$\therefore A \cup (B - A) \subset (A \cup B)$....(i)
Conversely,
Let $x \in (A \cup B)$
$\Rightarrow x \in A$ or $x \in B$
$\Rightarrow x \in A$ or $x \in (B - A)[\because B \subset (B - A)]$
$\Rightarrow x \in A \cup (B - A)$
$\therefore (A \cup B) \subset A \cup (B - A)$.........(ii)
From (i) and (ii), we get
$A \cup (B - A) = (A \cup B)$

16. Given, n(p) = 18, n(C) = 23, n(M) = 24, n(C ∩ M) = 13,
n(P ∩ C) = 12, n(P ∩ M) = 11 and n(P ∩ C ∩ M) = 6
    i. Total no. of students in the class
    = n(P ∪ C ∪ M)
    = n(P) + n(C) + n(M) - n(P ∩ C) - n(P ∩ M) - n(C ∩ M) + n(P ∩ C ∩ M)
    = 18 + 23 + 24 - 12 - 11 - 13 + 6 = 35
    ii. No. of students who took Mathematics but not Chemistry
    = n(M - C)
    = n(M) - n(M ∩ C)
    = 24 - 13 = 11
    iii. No. of students who took exactly one of the three subjects
    = n(P) + n(C) + n(M) - 2n(M ∩ P) - 2n(P ∩ C) - 2n(M ∩ C) + 3n(P ∩ C ∩ M)
    = 18 + 23 + 24 - 2 × 11 - 2 × 12 - 2 × 13 + 3 × 6
    = 65 - 22 - 24 - 26 + 18
    = 83 - 72 = 11
17. Here $A \cup X = B \cup X$ for some set X
$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$
$\Rightarrow A = (A \cap B) \cup (A \cap X)\ [\therefore A \cap (A \cup X) = A]$
$\Rightarrow A = (A \cap B) \cup \phi\ [\therefore A \cap X = \phi]$
$\Rightarrow A = A \cap B$
$\Rightarrow A \subset B...$(i)
Also $A \cup X = B \cup X$
$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$
$\Rightarrow (B \cap A) \cup (B \cap X) = B\ [\therefore B \cap (B \cup X) = B]$
$\Rightarrow (B \cap A) \cup \phi = B\ [\therefore B \cap X) = \phi]$
$\Rightarrow B \cap A = B$
$\Rightarrow B \subset A...$ (ii)
From (i) and (ii), we have
A = B.

18. Here$A \cup X = B \cup X$ for some set X

$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$

$\Rightarrow A = (A \cap B) \cup (A \cap X)[\because A \cap (A \cup X) = A]$

$\Rightarrow A = (A \cap B) \cup \phi[\therefore A \cap (A \cup X) = A]$

$\Rightarrow A = A \cap B$

$\Rightarrow A \subset B...(1)$

Also $A \cup X = B \cup X$

$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$

$\Rightarrow (B \cap A) \cup (B \cap X) = B[\because B \cap (B \cap X) = B]$

$\Rightarrow (B \cap A) \cup \phi = B[\therefore B \cap X = \phi]$

$\Rightarrow B \cap A = B$

. . . (ii)$\Rightarrow B \subset A...(ii)$

From (i) and (ii), we have

A=B

19. According to the question, we are given that,

U = {a, b, c, d, e, f} , A = {a, b,c}, B = {c, d, e, f} , C = {c, d, e} and D = {d, e, f}

i. A ∩ D = {a, b,c} ∩ {d, e, f} = ϕ

ii. A ∩ C = {a, b, c} ∩ {c, d, e} = {c}

iii. U ∩ D = {a, b, c, d, e, f} ∩ {d, e, f} = {d, e, f}

iv. $A \cup \phi$= {a, b, c} ∪ {} = {a,b,c}

v. U ∩ ϕ = {a, b, c, d, e, f} ∩ {} = ϕ

$(U \cap \phi)$'

= ϕ'

= U

vi. U ∪ A = {a, b, c, d, e, f} ∪ {a, ,b, c}

= {a, b, c, d, e, f}

= U

∴ (U ∪ A)' = ϕ

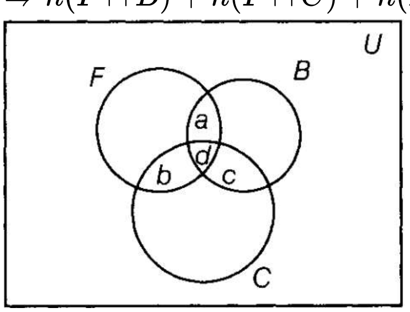
20. Suppose $n(F), n(B)\ and\ n(C)$ denote the number of men who received medals in Football, Basketball and Cricket, respectively. Then,

n(F) = 38, n(B) = 15, n(C) = 20, n($F \cup B \cup C$) = 58 and n($F \cap B \cap C$) = 3

$\because n(F \cup B \cup C)$= n(F) + n(B) + n(C) + $n(F \cap B \cap C)$ $-n(F \cap B) - n(F \cap C) - n(B \cap C)$

58 = 38 + 15 + 20 + 3$-n(F \cap B) - n(F \cap C) - n(B \cap C)$

$\Rightarrow n(F \cap B) + n(F \cap C) + n(B \cap C)$ = 76 - 58 = 18



Here, 'a' = the number of men who got medals in Football and Basketball only.

‘b' = the number of men who got medals in Football and Cricket only.

'c' = the number of men who got medals in Basketball and Cricket only.

'd ’= the number of men who got medals in all the three games.

Thus, d = $n(F \cap B \cap C)$ = 3

and $n(F \cap B) + n(F \cap C) + n(B \cap C)$ = 18

$\Rightarrow$ (a + d) + (b + d) + (c + d) = 18

$\Rightarrow$ a + b + c + 3d = 18

$\Rightarrow$ a + b + c+ 3(3) = 18 [Put d = 3, given]

$\therefore$ a + b + c = 9

Hence, people who got medals in exactly two of the three sports is 9.

21. Given,A = {4, 5, 6, 7, 8, 10}, B = {4, 5, 9} and C = {1, 4, 6, 9}
    i. Now, A∩B = {4, 5, 6, 7, 8, 10} ∩ {4, 5, 9} = {4, 5}

       $\therefore \quad \text{LHS} = (A \cap B) \cap C$

       = {4, 5} ∩ {1, 4, 6, 9}

       = {4}...........(i)

       Now, B∩C = {4, 5, 9} ∩ {1, 4, 6, 9} = {4, 9}

       $\therefore \text{RHS} = A \cap (B \cap C)$

       = {4, 5, 6, 7, 8, 10} ∩ {4, 9}

       = {4} ...(ii)

       From Eqs. (i) and (ii), we get

       LHS = RHS

       $\therefore \quad (A \cap B) \cap C = A \cap (B \cap C)$

       Hence verified.
    ii. Here, B ∩ C = {4,9}

       LHS = A ∪ (B ∩ C)

       = {4, 5, 6, 7, 8, 10} ∪ {4,9}

       $\Rightarrow$ LHS = {4,5, 6, 7,8,9,10}..........(iii)

       Now, A ∪ B = {4, 5, 6, 7, 8, 10} ∪{4, 5, 9}

       = {4, 5, 6, 7, 8, 9, 10}

       and A ∪ C= {4, 5, 6, 7, 8, 10} ∪ {1, 4, 6, 9}

       = $\{1, 4, 5, 6, 7, 8, 9, 10\}$

       RHS = (A ∪ B) ∩ (A ∪ C)

       = {4, 5, 6, 7, 8, 9, 10} ∩ {1, 4, 5, 6, 7, 8, 9, 10}

       = {4, 5, 6, 7, 8, 9, 10} ...(iv)

       From Eqs. (iii) and (iv), we get

       LHS = RHS = {4, 5, 6, 7, 8, 9, 10}

       $\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

       Hence verified.

iii. Now, B ∪ C = {4, 5, 9} ∪ {1, 4, 6, 9} = {1, 4, 5, 6, 9}

LHS = A ∩ (B ∪ C)

= {4, 5, 6, 7, 8, 10} ∩ {1, 4, 5, 6, 9} = {4, 5, 6} ...(v)

Now, A ∩ B = {4, 5, 6, 7, 8, 10} ∩ {4, 5, 9} = {4, 5}

and A ∩ C = {4, 5, 6, 7, 8, 10} ∩ {1, 4, 6, 9} = {4, 6}

RHS = $(A \cap B) \cup (A \cap C)$ = {4, 5} ∪ {4, 6}

= {4, 5, 6} ...(vi)

From Eqs. (v) and (vi), we get

LHS = RHS = {4, 5, 6}

$\therefore \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Hence Verified.