

Set Operations



Properties of Sets

Understanding the properties of sets is fundamental in set theory and helps in various mathematical operations. Here are some important properties of sets with examples:

1. Commutative Property

- **Union:** $A \cup B = B \cup A$
- **Intersection:** $A \cap B = B \cap A$

Example:

$$A = \{1, 2\}$$

$$B = \{2, 3\}$$

- $A \cup B = \{1, 2, 3\}$
- $B \cup A = \{1, 2, 3\}$
- $A \cap B = \{2\}$
- $B \cap A = \{2\}$

2. Associative Property

- **Union:** $(A \cup B) \cup C = A \cup (B \cup C)$
- **Intersection:** $(A \cap B) \cap C = A \cap (B \cap C)$

Example:

$$A = \{1, 2\}$$

$$B = \{2, 3\}$$

$$C = \{3, 4\}$$

- $(A \cup B) \cup C = \{1, 2, 3, 4\}$
- $A \cup (B \cup C) = \{1, 2, 3, 4\}$
- $(A \cap B) \cap C = \emptyset$
- $A \cap (B \cap C) = \emptyset$

3. Distributive Property

- **Intersection over Union:** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- **Union over Intersection:** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Example:

$$A = \{1, 2\}$$

$$B = \{2, 3\}$$

$$C = \{3, 4\}$$

- $A \cap (B \cup C) = A \cap \{2, 3, 4\} = \{2\}$
- $(A \cap B) \cup (A \cap C) = \{2\} \cup \emptyset = \{2\}$
- $A \cup (B \cap C) = A \cup \emptyset = \{1, 2\}$
- $(A \cup B) \cap (A \cup C) = \{1, 2, 3\} \cap \{1, 2, 4\} = \{1, 2\}$

4. Identity Property

- **Union with an empty set:** $A \cup \emptyset = A$
- **Intersection with an empty set:** $A \cap \emptyset = \emptyset$

Example:

$$A = \{1, 2\}$$

- $A \cup \emptyset = \{1, 2\}$
- $A \cap \emptyset = \emptyset$

5. Idempotent Property

- **Union:** $A \cup A = A$
- **Intersection:** $A \cap A = A$

Example:

$$A = \{1, 2\}$$

- $A \cup A = \{1, 2\}$
- $A \cap A = \{1, 2\}$

6. Complement Laws

- **Union with complement:** $A \cup A' = U$
- **Intersection with complement:** $A \cap A' = \emptyset$

Example:

$$U = \{1, 2, 3, 4\}$$

$$A = \{1, 2\}$$

$$A' = \{3, 4\}$$

- $A \cup A' = \{1, 2, 3, 4\} = U$
- $A \cap A' = \emptyset$

7. De Morgan's Laws

- **Union:** $(A \cup B)' = A' \cap B'$
- **Intersection:** $(A \cap B)' = A' \cup B'$

Example:

$$U = \{1, 2, 3, 4, 5\}$$

$$A = \{1, 2\}$$

$$B = \{2, 3\}$$

$$A' = \{3, 4, 5\}$$

$$B' = \{1, 4, 5\}$$

- $(A \cup B)' = \{1, 2, 3\}' = \{4, 5\}$
- $A' \cap B' = \{3, 4, 5\} \cap \{1, 4, 5\} = \{4, 5\}$
- $(A \cap B)' = \{2\}' = \{1, 3, 4, 5\}$
- $A' \cup B' = \{3, 4, 5\} \cup \{1, 4, 5\} = \{1, 3, 4, 5\}$

Problems

1. Prove that $A \cup B = B \cup A$ for $A = \{1, 2\}$ and $B = \{2, 3\}$.
2. Show that $A \cap B = B \cap A$ for $A = \{a, b\}$ and $B = \{b, c\}$.
3. Verify that $(A \cup B) \cup C = A \cup (B \cup C)$ for $A = \{1\}$, $B = \{1, 2\}$, and $C = \{2, 3\}$.
4. Prove that $(A \cap B) \cap C = A \cap (B \cap C)$ for $A = \{x, y\}$, $B = \{y, z\}$, and $C = \{z, w\}$.
5. Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ for $A = \{1, 2\}$, $B = \{2, 3\}$, and $C = \{3, 4\}$.
6. Verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ for $A = \{x\}$, $B = \{x, y\}$, and $C = \{y, z\}$.
7. Prove that $A \cup \emptyset = A$ and $A \cap \emptyset = \emptyset$ for $A = \{1, 2, 3\}$.
8. Show that $A \cup A = A$ and $A \cap A = A$ for $A = \{a, b, c\}$.
9. Verify De Morgan's law $(A \cup B)' = A' \cap B'$ for $U = \{1, 2, 3, 4\}$, $A = \{1\}$, and $B = \{2\}$.
10. Prove De Morgan's law $(A \cap B)' = A' \cup B'$ for $U = \{1, 2, 3, 4, 5\}$, $A = \{1, 2\}$, and $B = \{2, 3\}$.

Key

1. $A \cup B = B \cup A = \{1, 2, 3\}$
2. $A \cap B = B \cap A = \{b\}$
3. $(A \cup B) \cup C = \{1, 2\} \cup \{2, 3\} = \{1, 2, 3\}$, $A \cup (B \cup C) = \{1\} \cup \{1, 2, 3\} = \{1, 2, 3\}$
4. $(A \cap B) \cap C = \{y\} \cap \{z, w\} = \emptyset$, $A \cap (B \cap C) = \{x, y\} \cap \emptyset = \emptyset$
5. $(A \cap (B \cup C)) = \{1, 2\} \cap \{2, 3\} = \{2\}$