Sets Overview & Operations

Lecture Notes on Types of Sets

Empty Set

- **Definition**: A set that contains no elements. It is also known as the null set and is denoted by $\emptyset$ or $\{\}$.
- **Example**: The set of all natural numbers less than 1 is an empty set, $A = \{\}$.

Finite and Infinite Sets

- **Finite Set**:
  - **Definition**: A set with a countable number of elements.
  - **Example**: The set of vowels in the English alphabet, $V = \{a, e, i, o, u\}$.
- **Infinite Set**:
  - **Definition**: A set with an uncountable number of elements.
  - **Example**: The set of all natural numbers, $N = \{1, 2, 3, \ldots\}$.

Equal Sets

- **Definition**: Two sets are equal if they contain exactly the same elements. Denoted by $A = B$.
- **Example**: If $A = \{1, 2, 3\}$ and $B = \{3, 2, 1\}$, then $A = B$.

Subsets

- **Definition**: A set A is a subset of set B if every element of A is also an element of B. Denoted by $A \subseteq B$.
- **Example**: If $A = \{1, 2\}$ and $B = \{1, 2, 3, 4\}$, then $A \subseteq B$.
- **Proper Subset**:
  - **Definition**: A set A is a proper subset of set B if $A \subseteq B$ and $A \neq B$. Denoted by $A \subset B$.
  - **Example**: If $A = \{1, 2\}$ and $B = \{1, 2, 3\}$, then $A \subset B$.

Power Sets

- **Definition**: The set of all subsets of a set A, including A and the empty set. Denoted by $P(A)$.
- **Example**: If $A = \{1, 2\}$, then the power set of A is:

$$P(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$$

Universal Sets

- **Definition**: The set that contains all the objects under consideration, usually denoted by U.

- **Example**: If we are considering the set of all natural numbers, then the universal set could be $U = \{x \mid x \text{ is a natural number}\}$.

Detailed Examples

1. **Empty Set**:
   - **Example 1**: The set of all square roots of -1 in the set of real numbers, $\{x \mid x^2 = -1, x \in \mathrm{R}\} = \emptyset$.
   - **Example 2**: The set of students in a class who are older than 100 years, $\emptyset$.

2. **Finite and Infinite Sets**:
   - **Finite Set**:
     - The set of months in a year, $M = \{January, February, \ldots, December\}$.
     - The set of primary colors, $P = \{red, blue, yellow\}$.
   - **Infinite Set**:
     - The set of all even numbers, $E = \{2, 4, 6, 8, \ldots\}$.
     - The set of all points on a line, $L = \{(x, y) \mid y = mx + c\}$.

3. **Equal Sets**:
   - **Example 1**: $A = \{2, 4, 6\}$ and $B = \{4, 2, 6\}$, $A = B$.
   - **Example 2**: The set of letters in the word "silent" and the set of letters in the word "listen", $A = \{s, i, l, e, n, t\}$ and $B = \{l, i, s, t, e, n\}$, $A = B$.

4. **Subsets**:
   - **Example 1**: $A = \{2, 4\}$ and $B = \{2, 4, 6, 8\}$, $A \subseteq B$.
   - **Example 2**: The set of natural numbers N is a subset of the set of real numbers R, $\mathrm{N} \subseteq \mathrm{R}$.

5. **Power Sets**:
   - **Example 1**: If $B = \{a, b\}$, then $P(B) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$.
   - **Example 2**: If $C = \{1, 2, 3\}$, then $P(C) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$.

6. **Universal Sets**:
   - **Example 1**: For the set of all natural numbers, the universal set U could be all integers, $U = \mathrm{Z}$.
   - **Example 2**: In a classroom scenario, if we are only considering students, the universal set U could be the set of all students in the school.

These examples and definitions provide a comprehensive understanding of the different types of sets and their properties.