

Set Operations: Examples & Problems



Properties of Sets

1. Commutative Property

- **Union:** $A \cup B = B \cup A$
- **Intersection:** $A \cap B = B \cap A$

2. Associative Property

- **Union:** $(A \cup B) \cup C = A \cup (B \cup C)$
- **Intersection:** $(A \cap B) \cap C = A \cap (B \cap C)$

3. Distributive Property

- **Intersection over Union:** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- **Union over Intersection:** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

4. Identity Property

- **Union with an empty set:** $A \cup \emptyset = A$
- **Intersection with an empty set:** $A \cap \emptyset = \emptyset$

5. Idempotent Property

- **Union:** $A \cup A = A$
- **Intersection:** $A \cap A = A$

6. Complement Laws

- **Union with complement:** $A \cup A' = U$
- **Intersection with complement:** $A \cap A' = \emptyset$

7. De Morgan's Laws

- **Union:** $(A \cup B)' = A' \cap B'$
- **Intersection:** $(A \cap B)' = A' \cup B'$

Problems

1. Union of Sets

- Find the union of each of the following pairs of sets:
 - (i) $X = \{1, 3, 5\}$ and $Y = \{1, 2, 3\}$

- (ii) $A = \{a, e, i, o, u\}$ and $B = \{a, b, c\}$
- (iii) $A = \{x : x \text{ is a natural number and multiple of } 3\}$ and $B = \{x : x \text{ is a natural number less than } 6\}$
- (iv) $A = \{x : x \text{ is a natural number and } 1 < x \leq 6\}$ and $B = \{x : x \text{ is a natural number and } 6 < x < 10\}$
- (v) $A = \{1, 2, 3\}$ and $B = \emptyset$

2. Subset

- Let $A = \{a, b\}$ and $B = \{a, b, c\}$. Is $A \subseteq B$? What is $A \cup B$?

3. Union when $A \subseteq B$

- If $A \subseteq B$, what is $A \cup B$?

4. Union and Intersection of Multiple Sets

- If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$, and $D = \{7, 8, 9, 10\}$, find:
 - (i) $A \cup B$
 - (ii) $A \cup C$
 - (iii) $B \cup C$
 - (iv) $B \cup D$
 - (v) $A \cup B \cup C$
 - (vi) $A \cup B \cup D$
 - (vii) $B \cup C \cup D$

5. Intersection of Sets

- Find the intersection of each pair of sets of question 1 above:
 - (i) $X = \{1, 3, 5\}$ and $Y = \{1, 2, 3\}$
 - (ii) $A = \{a, e, i, o, u\}$ and $B = \{a, b, c\}$
 - (iii) $A = \{x : x \text{ is a natural number and multiple of } 3\}$ and $B = \{x : x \text{ is a natural number less than } 6\}$
 - (iv) $A = \{x : x \text{ is a natural number and } 1 < x \leq 6\}$ and $B = \{x : x \text{ is a natural number and } 6 < x < 10\}$
 - (v) $A = \{1, 2, 3\}$ and $B = \emptyset$

6. Multiple Set Operations

- If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$, and $D = \{15, 17\}$, find:
 - (i) $A \cap B$
 - (ii) $B \cap C$
 - (iii) $A \cap C \cap D$
 - (iv) $A \cap B \cap D$
 - (v) $B \cap D$
 - (vi) $A \cap (B \cup C)$
 - (vii) $A \cap D$
 - (viii) $A \cap (B \cup D)$
 - (ix) $(A \cap B) \cap (B \cup C)$
 - (x) $(A \cup D) \cap (B \cup C)$

7. Set Intersection with Conditions

- If $A = \{x : x \text{ is a natural number}\}$, $B = \{x : x \text{ is an even natural number}\}$, $C = \{x : x \text{ is an odd natural number}\}$, and $D = \{x : x \text{ is a prime number}\}$, find:
 - (i) $A \cap B$
 - (ii) $A \cap C$
 - (iii) $A \cap D$

- (iv) $B \cap C$
- (v) $B \cap D$
- (vi) $C \cap D$

Key

1. Union of Sets

- (i) $X \cup Y = \{1, 2, 3, 5\}$
- (ii) $A \cup B = \{a, b, c, e, i, o, u\}$
- (iii) $A \cup B = \{1, 2, 3, 4, 5, 6, 9, 12, 15, \dots\}$
- (iv) $A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$
- (v) $A \cup B = \{1, 2, 3\}$

2. Subset

- $A \subseteq B, A \cup B = \{a, b, c\}$

3. Union when $A \subseteq B$

- $A \cup B = B$

4. Union and Intersection of Multiple Sets

- (i) $A \cup B = \{1, 2, 3, 4, 5, 6\}$
- (ii) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- (iii) $B \cup C = \{3, 4, 5, 6, 7, 8\}$
- (iv) $B \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$
- (v) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- (vi) $A \cup B \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- (vii) $(B \cup C \cup D) = \{3, 4, 5, 6, 7, 8, 9, 10\}$