Sets Overview & Operations

Lecture Notes on Sets

Introduction to Sets

Definition of Sets

A set is defined as a well-defined collection of distinct objects, considered as an object in its own right. For example, the numbers 1, 2, and 3 are distinct objects when considered separately, but when they are considered collectively as the set $\{1, 2, 3\}$, they form a single object.

Examples of Sets in Mathematics and Everyday Life

- **Mathematics**:
  - The set of natural numbers: $N = \{1, 2, 3, \ldots\}$
  - The set of prime numbers less than 10: $P = \{2, 3, 5, 7\}$
  - The set of solutions to the equation $x^2 - 4 = 0$: $S = \{-2, 2\}$
- **Everyday Life**:
  - A collection of fruits in a basket: $\{apple, orange, banana\}$
  - A set of students in a class: $\{John, Mary, Ahmed\}$

Historical Context: Georg Cantor

Georg Cantor (1845-1918) was a German mathematician who founded set theory. He introduced the concept of sets while working on problems related to trigonometric series. Cantor's work laid the foundation for modern mathematics, particularly through his development of the theory of infinite sets and his introduction of the notion of cardinality, which measures the size of sets. Despite initial resistance from some mathematicians of his time, Cantor's theories have become fundamental in mathematics .

Representation of Sets

Roster or Tabular Form

In the roster or tabular form, a set is defined by listing all its elements, separated by commas, within curly braces. The order of elements in the set does not matter, and each element is listed only once.

- **Example**: The set of vowels in the English alphabet is represented as $V = \{a, e, i, o, u\}$.

Set-Builder Form

In the set-builder form, a set is defined by a property that its members must satisfy. This form is useful when dealing with large or infinite sets.

- **Example**: The set of all x such that x is a natural number less than 5 is represented as $A = \{x \mid x \in \mathrm{N}, x < 5\}$. This is read as "A is the set of all x such that x is a natural number and x is less than 5."

Detailed Concepts

1. **Roster Form**:
    - List all elements within curly braces.
    - **Examples**:
        - Set of all even positive integers less than 7: $\{2, 4, 6\}$
        - Set of all vowels in the English alphabet: $\{a, e, i, o, u\}$
        - Set of odd natural numbers: $\{1, 3, 5, \ldots\}$

2. **Set-Builder Form**:
    - Use a variable followed by a colon and a property that the elements of the set must satisfy.
    - **Examples**:
        - Set of all x such that x is a vowel in the English alphabet: $V = \{x \mid x \text{ is a vowel in the English alphabet}\}$
        - Set of all positive integers less than 10: $A = \{x \mid x \in \mathrm{N}, 3 < x < 10\}$
        - Set of all x such that x is a natural number which divides 42: $A = \{x \mid x \in \mathrm{N}, x \mid 42\}$

By using these representations, sets can be clearly defined and understood in both mathematical and practical contexts .