Solution

SEQUENCE AND SERIES

Class 11 - Mathematics

1. Let the rational number S be $0.4\overline{23}$
   $\because S = 0.4\overline{23}$ = 0.4 + 0.023 + 0.00023 + 0.0000023 + ... ∞
   $\Rightarrow$ S = 0.4 + 0.023 [1 + 10^{-2} + 10^{-4} + ... ∞]
   Clearly, S is a geometric series with the first term, a being 1 and the common ratio, r being 10^{-2}
   Using the formula of sum, we can write,
   $\therefore$ S = 0.4 + 0.023$\left[\frac{1}{1-10^{-2}}\right]$
   $\Rightarrow$ S = 0.4 + $\frac{2.3}{99}$
   $\Rightarrow S = \frac{419}{990}$
2. Given that 1 + 3 + 9 + 27 + ...to 7 terms
   Here, a = 1 and r = $\frac{3}{1}$ = 3
   n = 7 terms
   $S_n = \frac{3^7-1}{3-1} \Rightarrow S_n = \frac{2187-1}{3-1} \Rightarrow S_n = \frac{2186}{2} \Rightarrow S_n = 1093$
   Therefore, the sum of nth term of G.P is 1093.
3. $S_1, S_2, ..., S_n$ are the sum of n terms of G.P. a = 1, r = 1, 2, 3, ..., n
   Then, $S_1 + S_2 + 2S_3 + 3S_4 + ... + (n - 1)S_n$
   $\frac{1(1^n-1)}{1-1} + \frac{1(2^n-1)}{2-1} + \frac{2(3^n-1)}{3-1}$ + ... + (n - 1) 1$\left(\frac{1^n-1}{1-1}\right)$
   = 2^n - 1 + 23^n - 1 + 3.4^n - 1 + ...
   = $2^n + 3^n + 4^n + ... + n^n$
   Hence it is proved.
4. Given: -8 and -2.
   Geometric mean of two numbers = $\sqrt{ab}$
   = $\sqrt{-8 \times -2} = \sqrt{16} = \pm 4$
   Therefore, the required geometric mean between -8 and -2 is -4.
5. $\sum\limits_{n=2}^{10} 4^n$
   Expanding the given series as,
   = $4^2 + 4^3 + 4^4 + ... + 4^{10}$
   a = 4^2, r = $\frac{4^3}{4}$ = 4, n = 9
   $s_{10} = \frac{a\left(r^9-1\right)}{1-r}$
   = $\frac{1}{3}$ [4^{11} - 16]
   = $\frac{1}{3}$ [4^{11} - 16]
   = $\frac{16}{3}$ [4^9 - 1]
6. Let the G.M. between 2 and 8 be G.
   Then, 2, G, and 8 are in G.P.
   $\therefore G^2 = 2 \times 8$
   $\Rightarrow G^2 = 16$
   $\Rightarrow G = \pm\sqrt{16}$
   $\Rightarrow G = \pm 4$
7. Clearly, the given series is a geometric series in which a = 2, r = 3 > 1 and l = 4374.
   $\therefore$ the required sum = $\frac{(lr-a)}{(r-1)} = \frac{(4374\times3-2)}{(3-1)} = \frac{13120}{2}$ = 6560.
   Therefore, the required sum of the given series is 6560
8. Given that: $\frac{a}{1-r}$ = 6, a = 2 and r = ?
   sum of the series in the exponent, $S_\infty = \frac{a}{1-r} = \frac{2}{1-r}$ = 6 $\Rightarrow$ r = $\frac{2}{3}$

Therefore, the common ratio r = $\frac{2}{3}$

9. Here, n = 20,

$\therefore$ a_{20} = (20 - 1) (2 - 20) (3 + 20)

= 19 × (-18) × (23) = -7866.

10. First, term, a = 1

Common ratio, r = $\frac{a_2}{a_1} = \frac{4}{1}$ = 4

Using the condition of the AP we get,

$\therefore$ 9th term = a_9 = $ar^{(9-1)}$ = $1(4)^8 = 4^8$ = 65536

Thus, the 9th term of the given GP is 65536

11. Let $\frac{a}{r}$, a, ar be first three terms of the given G.P.

$\frac{a}{r} + a + ar = \frac{39}{10}$ ...(i)

$(\frac{a}{r})(a)(ar)$ = 1 ...(ii)

From (ii) we obtain $a^3 = 1 \Rightarrow a = 1$ (considering real roots only)

Substituting a = 1 in equation (i), we obtain

$\frac{1}{\mathrm{r}} + 1 + \mathrm{r} = \frac{39}{10}$

$\Rightarrow 1 + \mathrm{r} + \mathrm{r}^2 = \frac{39}{10}\mathrm{r}$

$\Rightarrow 10 + 10r + 10r^2 - 39r = 0$

$\Rightarrow 10r^2 - 29r + 10 = 0$

$\Rightarrow 10r^2 - 25r - 4r + 10 = 0$

$\Rightarrow 5r(2r - 5)-2(2r - 5) = 0$

$\Rightarrow (5r - 2)(2r - 5) = 0$

$\Rightarrow \mathrm{r} = \frac{2}{5}$ or $\frac{5}{2}$

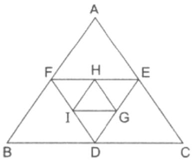
corresponding terms of the G.P

i. when r = $\frac{2}{5}$

$\Rightarrow$ $\frac{5}{2}$, 1, $\frac{2}{5}$

ii. when r = $\frac{5}{2}$

$\Rightarrow$ $\frac{2}{5}$, 1, $\frac{5}{2}$

12. 

Suppose △ABC be the given triangle having each side 18 cm. Suppose D, E, F be the midpoints of BC, CA, AB respectively to form △DEF. Suppose G, H, I be the midpoints of DE, EF and FD respectively to form △GHI.

We continue this process indefinitely. Then, we have

The sides of these triangle are 18 cm, 9 cm, $\frac{9}{2}$ cm, ..., and so on.

Sum of the areas of all triangles so formed

= $\frac{\sqrt{3}}{4}$ $\{(18)^2 + (9)^2 + (\frac{9}{2})^2 + (\frac{9}{4})^2 + ... \infty\}$ cm^2

= $\frac{\sqrt{3}}{4}\{324 + 81 + \frac{81}{4} + \frac{81}{16} + ... \infty\}$ cm^2

= $\frac{\sqrt{3}}{4} \cdot \frac{a}{(1-r)}$ cm^2, where a = 324 and r = $\frac{81}{324} = \frac{1}{4}$

= $\left\{ \frac{\sqrt{3}}{4} \times \frac{324}{\left(1-\frac{1}{4}\right)} \right\}$ cm^2 = $108\sqrt{3}$ cm^2

13. We have to prove: $\frac{1}{\log_a m}, \frac{1}{\log_b m}, \frac{1}{\log_c m}$ are in AP.

Here,it is given that: a, b, c are in GP

As, a, b, c are in GP

$\Rightarrow \frac{b}{a} = \frac{c}{b}$

Taking log both side

log $\frac{b}{a}$ = log $\frac{c}{a}$

$\Rightarrow$ log b - log a = log c - log b

$\Rightarrow$ 2log b = log a + log c

Dividing by log m,we get

$\Rightarrow 2(\frac{\log b}{\log m})$ = $\frac{\log a}{\log m} + \frac{\log c}{\log m}$ ($\frac{1}{\log_a m}$ = $\log_m$ a = $\frac{\log a}{\log m}$)

$\Rightarrow$ 2 $\log_m$ b = $\log_m$ a + $\log_m$ c

$\Rightarrow 2\frac{1}{\log_b m}$ = $\frac{1}{\log_a m} + \frac{1}{\log_c m}$

As we know that whenever a, b, c are in AP then 2b = a + c ......(i)

From the above equation (i), we can say that $\frac{1}{\log_a m}$, $\frac{1}{\log_b m}$, $\frac{1}{log_e m}$ are in AP.

Hence proved.

14. Here, it is given: a, b, c, d are in GP then

$\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

From the above, we can have the following conclusion

$\Rightarrow$ bc = ad ...(i)

$\Rightarrow$ b^2 = ac ...(ii)

$\Rightarrow$ c^2 = bd ...(iii)

Considering $\left(\frac{1}{a^2+b^2}, \frac{1}{b^2+c^2}, \frac{1}{c^2+d^2}\right)$

= $\frac{1}{a^2+b^2} \times \frac{1}{b^2+c^2}$ = $\frac{1}{a^2c^2+a^2d^2+b^2c^2+b^2d^2}$

= $\frac{1}{(ac)^2+(ad)^2+(bc)^2+(bd)^2}$

From eqn.(i), (ii) and (iii) ,we get

= $\frac{1}{\left(b^2\right)^2+(bc)^2+(bc)^2+\left(c^2\right)^2}$

= $\frac{1}{b^4+2b^2c^2+c^4}$

= $\frac{1}{\left(b^2+c^2\right)^2}$

From the above equation, we can say that $\left(\frac{1}{a^2+b^2}, \frac{1}{b^2+c^2}, \frac{1}{c^2+d^2}\right)$ are in GP.

Therefore, the following terms $\left(\frac{1}{a^2+b^2}, \frac{1}{b^2+c^2}, \frac{1}{c^2+d^2}\right)$ are in GP.

15. 12 + 8i, 11 + 6i, 10 + 4i, ...

This is an A.P.

Here, we have

a = 12 + 8i

d = (11 + 6i - 12 - 8i)

= (-1 - 2i)

Let the real term be a_n = a + (n - 1) d

a_n = (12 + 8i)+(n - 1)(-1 - 2i)

= (12 + 8i)+(-n + 1 - 2in + 2i)

= 12 + 8i - n + 1 - 2in + 2i

= (13 - n) + (8 - 2n + 2) i

= (13 - n)+(10 - 2n) i

a_n has to be real.

$\therefore$ (10 - 2n) = 0

$\Rightarrow$ n = 5

Hence, 5th term is purely real

Now, to find purely imaginary term of this A.P., real part have to be zero

$\therefore$ 13 - n = 0

⇒ n = 13

Hence, 13th term is purely imaginary.

16. Let the three numbers in GP be $\frac{a}{r}$, a, ar.

Sum of three numbers = 52 [given]

$\Rightarrow$ $\frac{a}{r}$ + a + ar = 52

$\Rightarrow$ a $\left(\frac{1}{r} + 1 + r\right)$ = 52 ...(i)

And sum of product in pair = 624

$\Rightarrow \frac{a}{r} \times$ a + a $\times$ ar + $\frac{a}{r} \times$ ar = 624

$\Rightarrow$ a^2 $\left(\frac{1}{r}+r+1\right)$= 624 ...(ii)

On dividing Eqs. (ii) by (i), we get

a = $\frac{624}{52} \Rightarrow$ a = 12

On putting a = 12 in Eq. (i), we get

12 ($\frac{1}{r}$ + r + 1) = 52

$\Rightarrow \frac{r^2+r+1}{r} = \frac{52}{12} \Rightarrow \frac{r^2+r+1}{r} = \frac{13}{3}$

$\Rightarrow$ 3r^2 +3r + 3 = 13r

$\Rightarrow$ 3r^2 - 10r + 3 = 0

$\Rightarrow$ (3r - 1) (r - 3) = 0

$\Rightarrow$ r = $\frac{1}{3}$ or r = 3

When r = $\frac{1}{3}$, then numers are $\frac{12}{\frac{1}{3}}$, 12, 12 $\times \frac{1}{3}$ i.e., 36, 12, 4.

When r = 3, then numbers are $\frac{12}{3}$, 12, 12 $\times \frac{1}{3}$ .i.e, 4, 12, 36.

17. $\frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{m}{n}$

$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$

by C and D

$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{m+n}{m-n}$

$\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{m+n}{m-n}$

$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{m+n}}{\sqrt{m-n}}$

by C and D

$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$

Sq both side

$\frac{a}{b} = \frac{m+n+m-n+2\sqrt{m^2-n^2}}{m+n+m-n-2\sqrt{m^2-n^2}}$

$\frac{a}{b} = \frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$

18. a + b = 6 $\sqrt{ab}$

$\frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$

by C and D

$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3+1}{3-1}$

$\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{2}{1}$

$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}}{1}$

again by C and D

$\frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$

$\frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$

$\frac{a}{b} = \frac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)^2}$ (on squaring both sides)

$\frac{a}{b} = \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$

$\frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$

a : b = (3 + 2$\sqrt{2}$) : (3 - 2$\sqrt{2}$)

19. Let G. P. be a, ar, ar^2, …..

Where r < 1

$S = \frac{a(1-r^n)}{1-r}$

$R = \frac{1}{a} + \frac{1}{ar} + - - + n$

$= \frac{\frac{1}{a}\left[\left(\frac{1}{r}\right)^{n} - 1\right]}{\frac{1}{r} - 1}$ $[\because r < 1 \text{ then } \frac{1}{r} > 1$

$= \frac{1}{a} \cdot \frac{1 - r^n}{r^n} \cdot \frac{r}{1 - r}$

$= \frac{1 - r^n}{ar^{n-1}(1 - r)}$

P = a.ar.ar^2 ... ar^{n-1}

= an.r$^{1+2+----+(n-1)}$

$= a^n . r^{\frac{n(n-1)}{2}}$

$= a^n . r^{\frac{n(n-1)}{2}}$

L.H.S. = P^{2}R^n

$= a^{2n} . r^{n(n-1)} \cdot \frac{(1 - r^n)^n}{ar^{n-1}(1 - r)^n}$

= S^n

P^2 = $\left(\frac{S}{R}\right)^n$

Hence proved.