Geometric Progression

Relationship between A.M. and G.M.:
Let A and G be the A.M. and G.M. of given positive real numbers a and b, then:

$$A = \frac{a+b}{2} \quad \text{and} \quad G = \sqrt{ab}$$

Thus, we have:

$$\begin{aligned} A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{a+b-2\sqrt{ab}}{2} \\ = \frac{(\sqrt{a}-\sqrt{b})^2}{2} \geq 0 \end{aligned}$$

Thus, we see that $A \geq G$.

Sum of First n Natural Numbers:

$$\Sigma n = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

Sum of the Sequences of the First n Natural Numbers:

$$\Sigma n^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of the Cubes of the First n Natural Numbers:

$$\Sigma n^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$