Geometric Progression

Geometric Means: When three quantities are in G.P., the middle one is called the Geometric Mean (G.M.) between the other two.

Thus, if a, G, b are in G.P., then G is the G.M. between a and b.
If a, G, b are in G.P., then:

$$\frac{G}{a} = \frac{b}{G} \quad [\text{since each is the common ratio}]$$

$$G^2 = ab \quad \Rightarrow \quad G = \sqrt{ab}$$

An Important Result:
The product of n G.M.'s between a and b is equal to the n-th power of the G.M. between a and b.

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n Geometric Means:
Let $G_1, G_2, G_3, \ldots, G_n$ be the n G.M.'s between a and b, then the total number of terms is $n+2$ and the number a and b.

$$n+2 \text{ terms} \quad \Rightarrow \text{last term} = b$$

If r is the common ratio,

$$b = ar^{n+2-1} = ar^{n+1}$$

Thus,

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$$

$$G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$
