Geometric Progression

Let's work through a few examples to better understand how to solve problems involving a Geometrical Progression (GP).

Example 1: Finding the nth Term of a GP

Problem: Given a GP where the first term $a = 5$ and the common ratio $r = 3$, find the 6th term of the sequence.

Solution:

The formula for the n-th term of a GP is:

$$T_n = a \cdot r^{n-1}$$

For $n = 6$:

$$T_6 = 5 \cdot 3^{6-1} = 5 \cdot 3^5 = 5 \cdot 243 = 1215$$

So, the 6th term of the sequence is **1215**.

Example 2: Sum of the First n Terms of a GP

Problem: Consider a GP where the first term $a = 4$ and the common ratio $r = 2$. Find the sum of the first 5 terms of the sequence.

Solution:

The formula for the sum of the first n terms of a GP is:

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad (\text{if } r \neq 1)$$

For $n = 5$, $a = 4$, and $r = 2$:

$$S_5 = \frac{4(1 - 2^5)}{1 - 2} = \frac{4(1 - 32)}{-1} = \frac{4 \cdot (-31)}{-1} = 124$$

So, the sum of the first 5 terms is **124**.

Example 3: Sum of an Infinite GP

Problem: Find the sum of the infinite GP with first term $a = 6$ and common ratio $r = \frac{1}{2}$.

Solution:

The formula for the sum of an infinite GP is:

$$S_\infty = \frac{a}{1-r} \quad \text{if } |r| < 1$$

Since $|r| = \frac{1}{2}$ (which is less than 1), the formula is valid.

$$S_\infty = \frac{6}{1-\frac{1}{2}} = \frac{6}{\frac{1}{2}} = 6 \times 2 = 12$$

So, the sum of the infinite series is **12**.

Example 4: Finding the Common Ratio

Problem: If the first term of a GP is $a = 8$ and the 4th term is 512, find the common ratio r.

Solution:

The formula for the n-th term is:

$$T_n = a \cdot r^{n-1}$$

For the 4th term:

$$T_4 = a \cdot r^{4-1} = 512$$

Substituting $a = 8$ and $T_4 = 512$:

$$8 \cdot r^3 = 512$$

Dividing both sides by 8:

$$r^3 = 64$$

Taking the cube root of both sides:

$$r = \sqrt[3]{64} = 4$$

So, the common ratio is **4**.

Let me know if you want to try more examples or need further explanation!