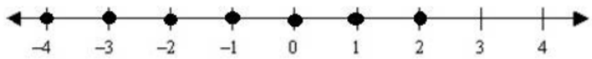
Solution

LINEAR INEQUALITY

Class 11 - Mathematics

1. Observe that the numerator is |x – 2| we know that modules function is always positive so the numerator is positive.
   Hence for $\frac{|x-2|}{(x-2)}$ to be a negative quantity the denominator (x – 2) has to be negative
   That is x – 2 should be less than 0
   ⇒ x – 2 < 0
   ⇒ x < 2
   Hence x should be less than 2 for $\frac{|x-2|}{(x-2)}$ < 0
   Hence x ∈ (-∞, 2)
   Hence the solution set for $\frac{|x-2|}{(x-2)} < 0$ is (-∞, 2)
2. Given, 5x + 2 < 17
   Subtracting 2 from both the sides in the above equation,
   ⇒ 5x + 2 – 2 < 17 – 2
   ⇒ 5x < 15
   Dividing both the sides by 5 in the above equation,
   ⇒ $\frac{5x}{5} < \frac{15}{5}$
   ⇒ x < 3
   Since x is an integer.
   Therefore, possible values of x can be
   x = {…, -2, -1, 0, 1, 2}
   

3. We are given 30 x < 200
   or $\frac{30x}{30} < \frac{200}{30}$
   i.e., x < 20 / 3.
   x < 6.66 for x ∈R
   When x ∈ Natural number,
   In this case, the following values of x make the statement true.
   1, 2, 3, 4, 5, 6.
   The solution set of the given linear inequality is {1, 2, 3, 4, 5, 6}.
4. Given, $\frac{3}{x-2} < 1$
   ⇒ $\frac{3}{x-2} - 1 < 0$
   ⇒ $\frac{3-x+2}{x-2} < 0$
   ⇒ $\frac{5-x}{x-2} < 0$
   ⇒ {5 -x < 0 & x - 2 > 0} or {5 -x > 0 & x - 2 < 0} [$\frac{a}{b}$ < 0 ⇔ {a < 0 & b > 0} or {a > 0 & b < 0}]
   ⇒ {-x < -5 & x > 2} or {-x > -5 & x < 2}
   ⇒ {x > 5 & x > 2} or {x < 5 & x < 2}
   ⇒ {x > 5} or {x < 2}
   ⇒ (-∞, 2) or (5, ∞)
   ⇒ (-∞, 2) ∪ (5, ∞)
   Hence solution of $\frac{3}{x-2} < 1$ is x ∈ (-∞, 2) ∪ (5, ∞)
5. Given, 5x + 2 < 17
   Subtracting 2 from both the sides in above equation
   ⇒ 5x + 2 – 2 < 17 – 2
   ⇒ 5x < 15
   Dividing both the sides by 5 in above equation
   ⇒ $\frac{5x}{5} < \frac{15}{5}$
   ⇒ x < 3

Therefore, x ∈ (-∞, 3)

6. We have, 2x + 6 ≥ 0
$\Rightarrow$ 2 x ≥ -6
$\Rightarrow$ x ≥-3
$\Rightarrow$x ∈ $[-3, \infty)$ ....(i)
Also, 4x - 7 < 0
$\Rightarrow$ 4x < 7
$\Rightarrow$ x < $\frac{7}{4}$
$\Rightarrow x \in \left(-\infty, \frac{7}{4}\right)$ ....(ii)
Thus, the solution of the given inequations is the intersection of (i) and (ii) is:
$[-3, \infty) \cap \left(-\infty, \frac{7}{4}\right)$ = $\left[-3, \frac{7}{4}\right)$
Thus, the solution of the given inequations is $\left[-3, \frac{7}{4}\right)$.
7. We have 24x < 100
$\Rightarrow \quad \frac{24x}{24} < \frac{100}{24}$ [dividing both sides by 24]
$\Rightarrow \quad x < \frac{25}{6}$
When x is a natural number, then solutions of the inequality are given by $x < \frac{25}{6}$ i.e., all natural numbers x which are less than $\frac{25}{6}$.
Hence, the solution set is {1, 2, 3, 4}
8. Given 12x < 50
$\Rightarrow \frac{12x}{12} < \frac{50}{12}$ [divide both sides by 12]
$\therefore x < \frac{25}{6}$
x ∈ R
When x is a real number, the solumımtion of the given inequation is $\left(-\infty, \frac{25}{6}\right)$.
9. Given:
$\frac{|x-3|}{x-3} < 0, x \in R$
|x - 3| < 0
The above condition can’t be true because the absolute value cannot be less than 0
but, it is possible only when denominator x-3 is negative so,
x - 3 < 0 $\Rightarrow$ x < 3
Therefore,
Solution set is; x ∈ (-∞, 3)
10. We have, 24x < 100
$\Rightarrow \quad \frac{24x}{24} < \frac{100}{24}$ [dividing both sides by 24]
$\Rightarrow \quad x < \frac{25}{6}$
When x is an integer.
Hence the solution set of inequality is {...., -4, -3, -2, -1, 0, 1, 2, 3, 4}
11. When,
|x - 2| ≤ 1
Then,
x – 2 ≤ -1 and x -2 ≥ 1
Now when,
x – 2 ≤ - 1
Adding 2 to both the sides in above equation
==> x – 2 + 2 ≤ -1 + 2
==> x ≤ 1
Now when,
x – 2 ≥ 1
Adding 2 to both the sides in above equation
==> x – 2 + 2 ≥ 1 + 2
==> x ≥ 3

For $|x - 2| \geq 1$ <==> $x \leq 1$ or $x \geq 3$

When,

$|x - 2| \leq 3$

Then,

$x - 2 \geq -3$ and $x - 2 \leq 3$

Now when,

$x - 2 \geq -3$

Adding 2 to both the sides in above equation

==> $x - 2 + 2 \geq -3 + 2$

==> $x \geq -1$

Now when,

$x - 2 \leq 3$

Adding 2 to both the sides in above equation

==> $x - 2 + 2 \leq 3 + 2$

==> $x \leq 5$

For $|x - 2| \leq 3$: $x \geq -1$ or $x \leq 5$

Combining the intervals:

$x \leq 1$ or $x \geq 3$ and $x \geq -1$ or $x \leq 5$

Merging the overlapping intervals:

$-1 \leq x \leq 1$ and $3 \leq x \leq 5$

Therefore,

$x \in [-1, 1] \cup [3, 5]$

12. Let x marks be scored by Tanvy in her last paper.

    It is given that Tanvy scored 89, 93, 95 and 91 marks in the first 4 papers.

    To receive grade A, she must obtain an average of 90 marks or more.

    Therefore, the average of there marks must more than equal to 90

    $\frac{89+93+95+91+x}{5} \geq 90$

    Multiplying both the sides by 5 in the above equation

    $\Rightarrow \left(\frac{89+93+95+91+x}{5}\right)(5) \geq 90(5)$

    $\Rightarrow 368 + x \geq 450$

    Subtracting 368 from both the sides in the above equation

    $\Rightarrow 368 + x - 368 \geq 450 - 368$

    $\Rightarrow x \geq 82$

    Therefore, Tanvy should score a minimum of 82 marks in her last paper to get grade A in the course.

13. Here $\frac{(2x-1)}{3} \geqslant \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

    $\Rightarrow \frac{2x}{3} - \frac{1}{3} \geqslant \frac{3x}{4} - \frac{2}{4} - \frac{2}{5} + \frac{x}{5}$

    $\Rightarrow \frac{2x}{3} - \frac{3x}{4} - \frac{x}{5} \geqslant \frac{-2}{4} - \frac{2}{5} + \frac{1}{3}$

    $\Rightarrow \frac{40x-45x-12x}{60} \geqslant \frac{-30-24+20}{60}$

    $\Rightarrow \frac{-17x}{60} \geqslant \frac{-34}{60}$

    Multiplying both sides by 60, we have

    $-17x \geqslant -34$

    Dividing both sides by -17, we have

    $\frac{-17x}{-17} \leqslant \frac{-34}{-17}$

    $\Rightarrow x \leqslant 2$

    Thus the solution set is $(-\infty, 2]$

14. Let the length of the shortest side be x cm.

    Then length of longest side = 3x cm

    length of third side = (3x - 2)cm

    Perimeter of triangle = x + 3x + 3x - 2

    = (7x - 2)cm

    Now $7x - 2 \geqslant 61$

$\Rightarrow 7x \geqslant 61+2 \Rightarrow 7x \geqslant 63 \Rightarrow x \geqslant 9$

Thus the minimum length of shortest side = 9 cm

15. Here 3x + 8 > 2

$\Rightarrow 3x > 2-8 \Rightarrow 3x > -6$

Dividing both sides by 3, we have

x > -2

(i) When x is an integer then values of x that make the statement true are -1, 0, 1, 2, 3, . . . The solution set of inequality is {-1, 0, 1, 2, 3, . . . }

(ii) When x is a real number. The solution set of inequality is $x \in (-2, \infty)$

16. The given system of linear inequalities is

2(2x + 3) -10 < 6(x - 2) ....(i)

and $\frac{2x-3}{4}+6 \geq 2+\frac{4x}{3}$ ...(ii)

From inequality (i), we get

2(2x + 3) - 10 < 6(x - 2)

$\Rightarrow$ 4x + 6 - 10 < 6x - 12

$\Rightarrow$ 4x - 4 < 6x - 12

$\Rightarrow$ 4x - 4 + 4 < 6x - 12 + 4 [adding 4 on both sides]

$\Rightarrow$ 4x < 6x - 8

$\Rightarrow$ 4x - 6x < 6x - 8 - 6x [subtracting 6x from both sides]

$\Rightarrow$ -2x < -8

$\Rightarrow$ 2x > 8 [dividing both sides by - 1 and then inequality sign will change]

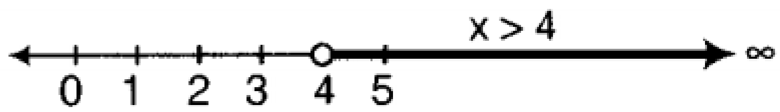
$\Rightarrow \frac{2x}{2} > \frac{8}{2}$ [dividing both sides by 2]

$\therefore \quad x > 4$ ...(iii)

Thus, any value of x greater than 4 satisfies the inequality.

$\therefore$ Solution set is $x \in (4, \infty)$

The representation of solution of inequality (i) is



From inequality (ii), we get

$\frac{2x-3}{4}+6 \geq 2+\frac{4x}{3} \Rightarrow \frac{2x-3+24}{4} \geq \frac{6+4x}{3}$

$\Rightarrow \frac{2x+21}{4} \geq \frac{6+4x}{3} \Rightarrow 3(2x+21) \geq 4(6+4x)$

$\Rightarrow 6x+63 \geq 24+16x$

$\Rightarrow -10x \geq -39 \Rightarrow 10x \leq 39$

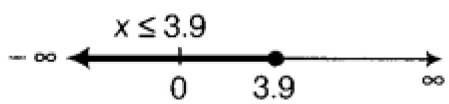
$\Rightarrow \frac{10x}{10} \leq \frac{39}{10}$

$\Rightarrow x \leq 3.9$ ...(iv)

Thus, any value of x less than or equal to 3.9 satisfies the inequality.

$\therefore$ Solution set is $x \in (-\infty, 3.9]$.

Its representation on number line is



From Eqs. (iii) and (iv), it is clear, that there is no common value of x, which satisfies both inequalities (iii) and (iv).

Hence, the given system of inequalities has no solution.

17. We have, $\frac{|x+3|+x}{x+2}$ > 1

$\Rightarrow \frac{|x+3|+x}{x+2}$ - 1 > 0

$\Rightarrow \frac{|x+3|+x-x-2}{x+2}$ > 0

$\Rightarrow \frac{|x+3|-2}{x+2}$ > 0

Let x + 3 = 0

$\Rightarrow$ x = - 3

$\therefore$ x = - 3 is a critical point.

So, here we have two intervals $(-\infty, -3)$ and $[-3, \infty)$

Case I: When - 3 $\leq$ x < ∞, then $|x+3|$= (x + 3)

$\therefore \frac{|x+3|-2}{x+2} > 0$

$\Rightarrow \frac{x+3-2}{x+2} > 0$

$\Rightarrow \frac{x+1}{x+2} > 0$

$\Rightarrow \frac{(x+1)(x+2)^2}{(x+2)} > 0 \times (x+2)^2$

$\Rightarrow$ (x + 1) (x + 2) > 0

Product of (x + 1) and (x + 2) will be positive, if both are of same sign.

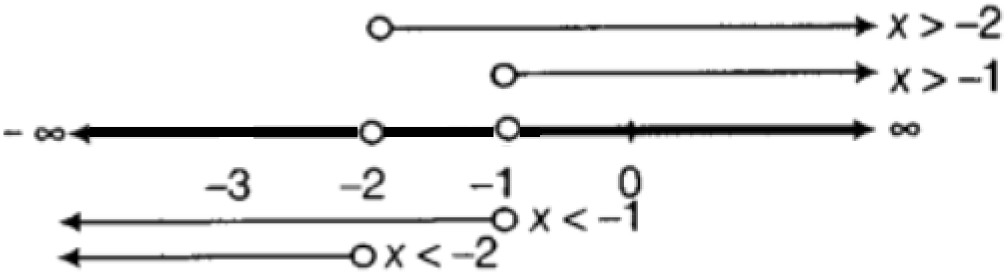
$\therefore$ (x + 1) > 0 and (x + 2) > 0

or (x + 1) < 0 and (x + 2) < 0

$\Rightarrow$ x > - 1 and x > - 2

or x < - 1 and x < - 2

On number line, these inequalities can be represented as,



Thus, - 1 < x < ∞ or - ∞ < x < - 2

But, here - 3 $\leq$ x < ∞

$\therefore$ - 1 < x < ∞ or - 3 $\leq$ x < - 2

Then, solution set in this case is

$x \in$ [- 3, - 2) $\cup$ (- 1, ∞)

Case II: When x < - 3, then $|x+3|$ = - (x + 3)

$\therefore \frac{|x+3|-2}{x+2} > 0$

$\Rightarrow \frac{-x-3-2}{x+2} > 0$

$\Rightarrow \frac{-(x+5)}{x+2} > 0$

$\Rightarrow \frac{x+5}{x+2} < 0$

$\Rightarrow \frac{(x+5)(x+2)^2}{x+2} < 0 \times (x+2)^2$

$\Rightarrow$ (x + 5) (x + 2) < 0

Product of (x + 5) and (x + 2) will be negative, if both are of opposite sign.

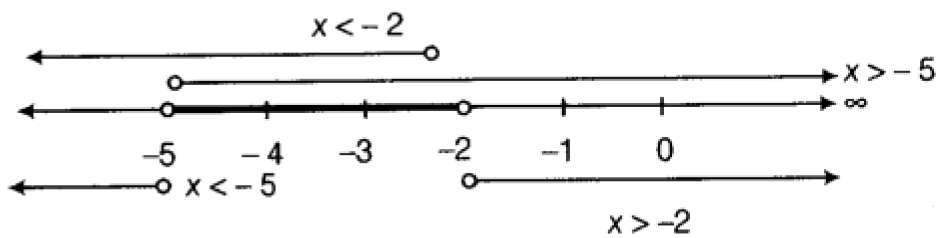
$\therefore$ (x + 5) > 0 and (x + 2) < 0

or (x + 5) < 0 and (x + 2) > 0

$\Rightarrow$ x > - 5 and x < - 2

or x < - 5 and x > - 2

On number line, these inequalities can be represented as,



Thus, - 5 < x < - 2 i.e., solution set in the case is $x \in$ (- 5, - 2).

On combining cases I and II, we get the required solution set of given inequality, which is

$x \in$ (- 5, - 2) $\cup$ (- 1, ∞)

18. The given system of linear inequalities is

- 2 - $\frac{x}{4} \geq \frac{1+x}{3}$ ... (i)

and 3 - x < 4 (x - 3) ... (ii)

From inequality (i), we get

$-2-\frac{x}{4} \geq \frac{1+x}{3}$

⇒ - 24 - 3x $\geq$ 4 + 4x [multiplying both sides by 12]

⇒ - 24 - 3x - 4 $\geq$ 4 + 4x - 4 [subtracting 4 from both sides]

⇒ - 28 - 3x $\geq$ 4x

⇒ - 28 - 3x + 3x $\geq$ 4x + 3x [adding 3x on both sides]

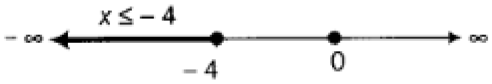
⇒ - 28 $\geq$ 7x

⇒ $-\frac{28}{7} \geq \frac{7x}{7}$ [dividing both sides by 7]

⇒ - 4 $\geq$ x or x $\leq$ - 4 ... (iii)

Thus, any value of x less than or equal to - 4 satisfied the inequality.

So, solution set is $x \in (-\infty, -4]$



From inequality (ii), we get

3 - x < 4 (x - 3)

⇒ 3 - x < 4x - 12

⇒ 3 - x + 12 < 4x - 12 + 12 [adding 12 on both sides]

⇒ 15 - x < 4x

⇒ 15 - x + x < 4x + x [adding x on both sides]

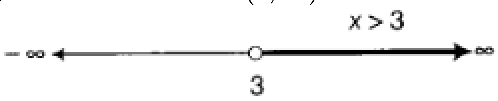
⇒ 15 < 5x

⇒ 3 < x [dividing both sides by 3]

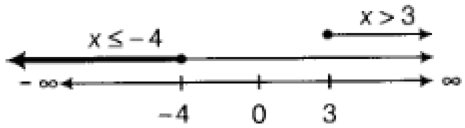
or x > 3 ... (iv)

Thus, any value of x greater than 3 satisfies the inequality.

So, the solution set is $x \in (3, \infty)$



The solution set of inequalities (i) and (ii) are represented graphically on number line as given below:



As no region is common, hence the given system has no solution.

19. Given:

$|x+a|+|x|>3, x \in R$

$|x+a|=-(x+a)$ or (x + a)

$|x|=-x$ or x

When $|x+a|=-(x+a)$ and $|x|=-x$

Then,

$|x+a|+|x|>3 \rightarrow$ -(x + a) + (-x) > 3

-x - a - x > 3

-2x - a > 3

Adding a on both the sides in above equation

-2x - a + a > 3 + a

-2x > 3 + a

Dividing both the sides by 2 in above equation

$\frac{-2x}{2} > \frac{3+a}{2}$

$-x > \frac{3+a}{2}$

Multiplying both the sides by -1 in the equation

$-x(-1) > \left(\frac{3+a}{2}\right)(-1)$

$x < -\left(\frac{3+a}{2}\right)$

Now, when, $|x+a|=-(x+a)$ and $|x|=x$

Then,

$|x+a|+|x|>3 \rightarrow$ -(x + a) + x > 3

-x - a + x > 3

-a > 3

In this case no solution for x.

Now when, $|x+a|=(x+a)$ and $|x|=-x$

Then,

$|x+a|+|x|>3 \rightarrow$ (x + a) + (-x) > 3

x + a - x > 3

a > 3

In this case no solution for x,

Now when,

$|x+a|=(x+a)$ and $|x|=x$

x + a + x > 3

2x + a > 3

Subtracting a from both the sides in above equation

2x + a - a > 3 - a

2x > 3 a

Dividing both sides by 2 in the above equation

$\frac{2x}{2}>\frac{3-a}{2}$

$x>\frac{3-a}{2}$

Therefore,

$x<-\left(\frac{3+a}{2}\right)$ or $x>\left(\frac{3-a}{2}\right)$