Saitechinfo NEET-JEE Academy

Limits Problems Explained with Examples

Limits help us understand the behavior of a function as the input approaches a specific value. Here are different types of problems with examples to illustrate key concepts.

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1. Basic Concept: Evaluating Directly

Example 1:

Evaluate $\lim_{x \to 2}(x^2 + 3x - 4)$.

Solution:

1. Substitute $x = 2$ directly into the function.

$$f(x) = x^2 + 3x - 4$$

$$f(2) = (2)^2 + 3(2) - 4 = 4 + 6 - 4 = 6$$

2. Therefore, $\lim_{x \to 2}(x^2 + 3x - 4) = 6$.

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2. Indeterminate Form: Simplify First

Example 2:

Evaluate $\lim_{x \to 2} \frac{x^2-4}{x-2}$.

Solution:

1. Substituting $x = 2$:

$$\frac{x^2 - 4}{x - 2} = \frac{4 - 4}{2 - 2} = \frac{0}{0}$$

   This is an indeterminate form.
2. Simplify the expression:

$$\frac{x^2 - 4}{x - 2} = \frac{(x - 2)(x + 2)}{x - 2}$$

   Cancel $(x - 2)$, assuming $x \neq 2$:

$$\frac{x^2 - 4}{x - 2} = x + 2$$

3. Substitute $x = 2$ into the simplified expression:

$$x + 2 = 2 + 2 = 4$$

4. Therefore, $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = 4$.

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3. Limit at Infinity

Example 3:

Evaluate $\lim_{x \to \infty} \frac{5x^2 + 3x + 2}{2x^2 + 7}$.

Solution:

1. Divide numerator and denominator by the highest power of x in the denominator (x^2):

$$\frac{5x^2 + 3x + 2}{2x^2 + 7} = \frac{5 + \frac{3}{x} + \frac{2}{x^2}}{2 + \frac{7}{x^2}}$$

2. As $x \to \infty$, terms like $\frac{1}{x}$ and $\frac{1}{x^2}$ approach 0:

$$\frac{5 + 0 + 0}{2 + 0} = \frac{5}{2}$$

3. Therefore, $\lim_{x \to \infty} \frac{5x^2 + 3x + 2}{2x^2 + 7} = \frac{5}{2}$.

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4. Trigonometric Limits

Example 4:

Evaluate $\lim_{x \to 0} \frac{\sin(x)}{x}$.

Solution:

1. This is a standard limit result:

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

Example 5:

Evaluate $\lim_{x \to 0} \frac{1 - \cos(x)}{x^2}$.

Solution:

1. Use the approximation for small x:

$$1 - \cos(x) \approx \frac{x^2}{2}$$

2. Substitute into the limit:

$$\lim_{x \to 0} \frac{1 - \cos(x)}{x^2} = \lim_{x \to 0} \frac{\frac{x^2}{2}}{x^2}$$
$$= \frac{1}{2}$$

3. Therefore, $\lim_{x \to 0} \frac{1-\cos(x)}{x^2} = \frac{1}{2}$.

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5. Special Exponential Limits

Example 6:

Evaluate $\lim_{x \to 0} \frac{e^x - 1}{x}$.

Solution:

1. Use the standard result:

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

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6. Limits Involving Piecewise Functions

Example 7:

Evaluate $\lim_{x \to 0^+} f(x)$ and $\lim_{x \to 0^-} f(x)$ for:

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Solution:

1. Right-hand limit ($x \to 0^+$): When $x \geq 0$, $f(x) = x^2$:

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x^2 = 0$$

2. Left-hand limit ($x \to 0^-$): When $x < 0$, $f(x) = -x$:

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} -x = 0$$

3. Since both limits are equal, $\lim_{x \to 0} f(x) = 0$.

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7. Squeeze Theorem

Example 8:

Evaluate $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$.

Solution:

1. The sine function is bounded:

$$-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$$

2. Multiply by x^2:

$$-x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2$$

3. As $x \to 0$, both $-x^2$ and x^2 approach 0:

$$\lim_{x\to 0} -x^2 = 0 \quad \text{and} \quad \lim_{x\to 0} x^2 = 0$$

4. By the Squeeze Theorem:

$$\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$$

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8. Limits Using L'Hôpital's Rule

Example 9:

Evaluate $\lim_{x\to 0} \frac{\ln(1+x)}{x}$.

Solution:

1. Substituting $x = 0$ gives $\frac{0}{0}$, an indeterminate form.
2. Apply L'Hôpital's Rule: Differentiate numerator and denominator:

$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = \lim_{x\to 0} \frac{\frac{1}{1+x}}{1}$$

3. Simplify:

$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = \frac{1}{1+0} = 1$$

These examples cover a wide range of limit problems using different techniques. Practice these types to strengthen your understanding of limits!