Complex Number Formulas

1. **Complex Number**: A number of the form $z = a + ib$, where a is the real part, b is the imaginary part, and i is the imaginary unit defined as $i^2 = -1$.
2. **Real Part** ($\Re(z)$): The real component of a complex number, $\Re(z) = a$, where $z = a + ib$.
3. **Imaginary Part** ($\Im(z)$): The imaginary component of a complex number, $\Im(z) = b$, where $z = a + ib$.
4. **Imaginary Unit** (i): Defined as $i^2 = -1$, the square root of -1.
5. **Modulus** ($|z|$): The magnitude of a complex number, defined as $|z| = \sqrt{a^2 + b^2}$ for $z = a + ib$.
6. **Argument** ($\arg(z)$): The angle θ made by the complex number in the complex plane with the positive real axis, defined as $\arg(z) = \tan^{-1}\left(\frac{b}{a}\right)$.
7. **Conjugate** ($\bar{z}$): The complex conjugate of $z = a + ib$, defined as $\bar{z} = a - ib$.
8. **Polar Form**: A representation of a complex number in terms of modulus and argument: $z = r(\cos\theta + i\sin\theta)$, where $r = |z|$ and $\theta = \arg(z)$.
9. **Exponential Form**: A compact form of the polar representation using Euler's formula: $z = re^{i\theta}$, where $r = |z|$ and $\theta = \arg(z)$.
10. **Euler's Formula**: $e^{i\theta} = \cos\theta + i\sin\theta$, a key relation connecting exponential, trigonometric, and complex numbers.
11. **Addition of Complex Numbers**: For two complex numbers $z_1 = a + ib$ and $z_2 = c + id$, their sum is $z_1 + z_2 = (a + c) + i(b + d)$.
12. **Multiplication of Complex Numbers**: For two complex numbers $z_1 = a + ib$ and $z_2 = c + id$, their product is $z_1 z_2 = (ac - bd) + i(ad + bc)$.
13. **Division of Complex Numbers**: For two complex numbers $z_1 = a + ib$ and $z_2 = c + id$, the quotient is $\frac{z_1}{z_2} = \frac{(a+ib)(c-id)}{c^2+d^2} = \frac{(ac+bd)+i(bc-ad)}{c^2+d^2}$.
14. **De Moivre's Theorem**: For any complex number in polar form, $z = r(\cos\theta + i\sin\theta)$, and integer n, $z^n = r^n\left(\cos(n\theta) + i\sin(n\theta)\right)$.
15. **Roots of Unity**: The solutions to $z^n = 1$ are given by $z_k = e^{2k\pi i/n}$, where $k = 0, 1, 2, \ldots, n-1$. These are equally spaced points on the unit circle in the complex plane.
16. **Quadratic Equations in Complex Numbers**: The roots of a quadratic equation $ax^2 + bx + c = 0$ are given by $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$, and if the discriminant $b^2 - 4ac < 0$, the solutions are complex.
17. **Complex Plane (Argand Diagram)**: A geometric representation of complex numbers, where the x-axis represents the real part, and the y-axis represents the imaginary part.
18. **Rotation Theorem**: Rotating a complex number z by an angle α is equivalent to multiplying it by $e^{i\alpha}$. That is, $z' = ze^{i\alpha}$.
19. **Distance Between Two Complex Numbers**: The distance between two complex numbers z_1 and z_2 is $|z_1 - z_2|$.
20. **Inverse of a Complex Number**: The inverse of a non-zero complex number $z = a + ib$ is $z^{-1} = \frac{1}{z} = \frac{a-ib}{a^2+b^2}$.